Optical pumping: measuring and observing two-quantum transitions

Objects of the experiment

- Observation of prohibited Zeeman transitions on the ground state of $^{87}$Rb
- Recording the power dependency

Principles

Optical pumping [1,2,3] permits spectroscopic analysis of atomic energy states in an energy range not accessible to direct optical observation.

In weak magnetic fields, the differences in the population number between the Zeeman levels in the ground state of $^{87}$Rb are extremely slight, as the energy interval is less than $10^{-8}$ eV. Optical pumping produces a population which deviates greatly from the thermal equilibrium population. To accomplish this, rubidium vapour is irradiated in an absorption cell with the circularly polarised component of the D$_1$ light from a rubidium lamp. The population of the Zeeman level depends on the polarity of the incident light. When the cell is irradiated with a high-frequency alternating magnetic field, we observe a change in the transparency of the rubidium vapour for rubidium-D$_1$ light.

A rubidium high-frequency lamp is used as the pumping light source. Rubidium atoms in a glass ampoule are excited in the electromagnetic field of an HF transmitter.

The combination of an interference filter, a polarisation filter and a quarter-wavelength plate separate the desired circularly polarised component of the D$_1$ line from the emission spectrum of the light source. Depending on the position of the quarter wavelength plate, we obtain either $\sigma^+$ or $\sigma^-$ polarisation.
A system of convex lenses focuses the pumping light on the centre of the absorption cell (also filled with rubidium vapour) and the transmitted component of the pumping light on a photodetector (cf. Fig. 3).

The Zeeman magnetic field is generated using Helmholtz coils. Depending on the polarity of the coil current, the field lines are oriented either parallel or anti-parallel to the optical radiation.

Using the high-frequency coil pair, it is possible to generate a high-frequency alternating field perpendicular to the Zeeman magnetic field. When its frequency corresponds to the energy difference of two adjacent Zeeman levels, transition between the levels can occur. The populations of the Zeeman levels, and thus the transparency of the rubidium vapour, change.

To determine the change in transparency, the intensity of the transmitted light is measured using a silicon photodetector. A current/voltage converter amplifies its output signal. The transmitted light is measured using a silicon photodetector. A system of convex lenses focuses the pumping light on the centre of the absorption cell (also filled with rubidium vapour).

**Physical principles**

In its ground state, rubidium, like all alkali metals, has a total spin of the electron shell with the spin quantum number \( J = \frac{1}{2} \). The ground state thus splits into two hyperfine states with the total angular momenta \( F = I + \frac{1}{2} \) and \( F = I - \frac{1}{2} \) respectively.

In the magnetic field, the hyperfine states are each split into \( 2F + 1 \) Zeeman levels with the magnetic quantum numbers \( m_F = -F, \ldots, F \). Fig. 2 shows an example of the level diagram for \(^{87}\text{Rb}\).

![Schematic representation of Zeeman levels in the ground state of \(^{87}\text{Rb}\)](image)

The energy \( E \) of the Zeeman levels can be calculated for the magnetic fields used here with the help of the Breit-Rabi formula [4,5]:

For \( F = I \pm \frac{1}{2} \)

\[
E(F,m_F) = -\frac{\Delta E}{2(2I+1)} + \mu_B g_I B m_F \pm \frac{\Delta E}{2} \left(1 + \frac{4m_F}{2I+1} \xi + \xi^2 \right)^{\frac{1}{2}}
\]

where \( \xi = \frac{g_I \mu_B - g_K \mu_K}{\Delta E} \)

\( F \): Total angular momentum

\( I \): Nuclear spin

\( J \): Angular momentum of the electron shell

\( m_F \): Magnetic quantum number of the total angular momentum \( F \)

\( g_I \): g-factor of the nucleus

\( g_J \): g-factor of the electron shell

\( \Delta E \): Hyperfine structure spacing

\( \mu_B \): Bohr magneton

\( \mu_K \): Nuclear magneton

\( B \): Magnetic flux density

When irradiated with \( \sigma^+ \) pumping light, the Zeeman levels within a hyperfine state which have positive quantum numbers \( m_F \) become enriched. The result is a population which deviates from the thermal equilibrium population.

The energy difference between adjacent \( m_F \) Zeeman levels in a magnetic field of 1 mT corresponds to the high-frequency range of about 7 MHz. When irradiated with a linearly polarized alternating magnetic field of the correct frequency, more transitions take place from a higher Zeeman level \( F \) to the next lowest level with -1 \( F \) than in the other direction.

This results in a change in the optical transmission.

With increasing power, a point is reached at which both levels are occupied equally and optical transparency no longer changes with increasing power. At this point the system is saturated.

In addition to normal transitions where \( \Delta m_F = \pm 1 \), at higher amplitudes of the alternating field it is possible for two field quanta to effectively be absorbed simultaneously, triggering a transition where \( \Delta m_F = \pm 2 \). This occurs at the arithmetic mean of the two \( \Delta m_F = \pm 1 \) transitions, meaning that the corresponding "two quantum" lines appear half way between the normal lines. Since two field quanta would have to be absorbed at the same time, the power dependency is different than for normal lines.

The frequency \( f \) of these transitions is

\[
f(m_F \leftrightarrow m_F - 2) = \frac{1}{2} \left[ \pm \frac{\mu_K g_I B}{\hbar} + \frac{\Delta E}{2\hbar} \left(1 + \frac{4m_F}{2I+1} \xi + \xi^2 \right)^{\frac{1}{2}} - \frac{4m_F}{2I+1} \xi + \xi^2 \left(1 + \frac{4m_F}{2I+1} \xi + \xi^2 \right)^{\frac{1}{2}} \right]
\]

(II)
Safety notes

Protecting individuals
Danger of scalding: hot water can leak from insecurely fastened or defective water tubing between the circulation thermostat and the absorption chamber:
- Use only silicon tubing of the specified diameter.
- Clamp the tubes in the holder between the Helmholtz coils and secure them against slippage.

Protecting the equipment
The absorption chamber is made of acrylic glass and can be destroyed by heat:
- Fill the absorption chamber with distilled water only.
- Do not heat the absorption chamber above 80°C.
- Never clean the absorption chamber with solvents.
The uniformity of the Helmholtz field is impaired if the Helmholtz coil cores become deformed:
- Protect Helmholtz coils from shocks or knocks.
The HF transmitter in the rubidium high-frequency lamp can be destroyed by excessive voltage levels:
- Only operate the rubidium high-frequency lamp with the optical pumping supply unit.

For best experiment results
The experiment setup is sensitive to interfering magnetic fields:
- Keep all power supplies and measuring instruments as far away from the experiment setup as possible.
- Remove ferromagnetic materials or devices which generate magnetic fields from the vicinity of the experiment setup.
- Use only lenses on brass stems (460 021 and 460 031).

Room lighting can drown out the measurement signal at the silicon photodetector. External light unnecessarily raises the DC component of the photodetector signal:
- Switch off the electric lighting in the room.
- Prevent the incidence of external light.
- Darken the experiment room.
- Turn the reflective side of the line filter so that it faces the rubidium high-frequency lamp.

The direction of flow of the heating water in the absorption chamber is determined by the experiment setup:
- Make sure the water inlets and outlets are connected in the proper direction.

High frequencies interfere with voltage-sensitive measuring instruments:
- Only use the rubidium high-frequency lamp when it is fully assembled.

Equipment list

1 Rubidium high-frequency lamp ..................... 558 823
1 Pair of Helmholtz coils on stand rider .......... 558 826
1 Absorption chamber with rubidium absorption cell .......... 558 833
1 Silicon photodetector ................................ 558 835
1 I/V converter for silicon photodetector .......... 558 836

1 Supply unit for optical pumping ................. 558 814
1 Function generator, 1 MHz - 12 MHz ............ 522 551
1 DC power supply, 0...±15 V .................... 521 45
1 Circulation thermostat, +30°C to +100°C ........ 666 768
1 Digital storage oscilloscope, e.g. ............... 575 294
1 Plug-in power unit, 9,2V-, regulated .......... 530 88
1 Digital/analog multimeter MetraHit Pro ........ 531 281
1 Two-way switch .................................. 504 48

1 Optical bench, standardised profile, 1m ........ 460 32
1 Line filter, 795 nm .................................. 468 000
1 Polarisation filter for red radiation ............. 472 410
1 Quarter-wavelength plate, 200 nm ............... 472 611
1 Lens on brass stem, f = +50 mm ................. 460 021
1 Lens on brass stem, f = +100 mm ............... 460 031
6 Optical riders 60/34 ............................... 460 370
1 Optical rider 95/50 ................................ 460 374
1 Silicone tube, 5 m long, 6.0x2.0 .................. 688 115
4 Connecting leads, black, 50 cm ............... 501 28
2 Connecting leads, black, 200 cm ............... 501 38
3 BNC cables, 1 m .................................. 501 02
1 BNC cable, 2 m .................................. 501 022
2 Canisters of water, pure, 5 l ..................... 675 3410

The following applies for the Helmholtz coils:

\[
B = \left( \frac{4}{5} \right)^{3/2} \frac{\mu_0 N}{r_{\text{eff}}} 
\]

(III)

where, for example, 
\( r_{\text{eff}} = 116 \text{ mm} \)
\( N=210 \)
results in a field
\( B= 1.205 \text{ mT for } I = 0.740 \text{ A} \)
Setup

Optical and electrical setup
- Set up the optical and magnetic components on the optical bench with standardised profile (460 32), as shown in Fehler! Verweisquelle konnte nicht gefunden werden. and Fig. 2.
- Connect the rubidium high-frequency lamp with the supply unit for optical pumping (558 814).
- Connect the Helmholtz coils and the multimeter (531 281) in series to the power supply (521 45).
- Insert the two-way switch (504 48) in the circuit to permit easy reversal of the magnetic field.
- Connect the output of the function generator (522 551) to the HF coils.
- Connect the photodetector output to channel I of the oscilloscope (575 294) via the I/V converter (558 836).
- Connect the sweep output, Sweep Out, on the function generator to channel II of the oscilloscope.

Warming up the system
- Using silicone tubing, set up a heating water circuit between the absorption chamber and the circulation thermostat (666 768) as shown in Fig. 2.
- Switch on the supply unit for optical pumping and set the operating current to about 0.8 A (cf. instruction sheet for the rubidium high-frequency lamp 558 823).
- Switch on the stabilised power supply.
- Wait at least 15 min. until the operating temperature is reached.

If the light output of the rubidium high-frequency lamp is unstable:
- Increase the operating current by about 0.1 A.
Initial optical adjustment
- Remove the optical riders with line filter, polarisation filter and quarter wavelength plate from the optical bench.
- Remove the absorption chamber from the stand rider for the Helmholtz coils.
- Hold a white piece of paper in place of the absorption cell at the midpoint between the Helmholtz coils.
- Move the lens (6) and the rubidium high-frequency lamp so that the smallest possible evenly illuminated light spot is obtained (cf. Fig. 3).
- Remove the optical rider with the silicon photodetector from the optical bench.
- Using the piece of paper, find the point with the smallest evenly illuminated light spot.
- Move lens (8) to improve the illumination (cf. Fig. 3).
- Set up the silicon photodetector at the point where the piece of paper is.

When the initial adjustment is complete:
- Set up the components that were previously removed on the optical bench again.

Finding the absorption signal
For $^{87}$Rb the frequencies of the Zeeman transitions in the ground state in a Helmholtz field of 1.2 mT (coil current 740 mA) are around 8.4 MHz. This can be scaled up or down accordingly for other currents. With careful adjustment for a fast sweep, the absorption can reach an amplitude of some 20 mV (higher signals could be achievable if the operating current of the rubidium lamp were increased, but this also would reduce the lifespan of the lamp): For slower measurements, the amplitude can be 100 mV as in the sample measurements.
- Set the polariisation filter to 0° and the quarter wavelength plate to +45° or -45°.
- Set the desired operating mode and frequency range on the function generator.
- Start the function generator by pressing the button labelled MANUAL.
- Vary the Helmholtz coil current until the maximum (negative) absorption signal appears on the oscilloscope.
- If necessary, set the toggle switch of the I/V converter to AC or use the offset potentiometer of the I/V converter to bring the signal back to the middle of the oscilloscope screen.
- Maximise the absorption signal by changing the operating parameters of the rubidium high-frequency lamp.

Settings
\begin{itemize}
  \item Polarisation filter:
    \begin{itemize}
      \item Angle: 0°
    \end{itemize}
  \item Quarter wavelength plate:
    \begin{itemize}
      \item Angle: +45° or -45°
    \end{itemize}
  \item I/V converter:
    \begin{itemize}
      \item Toggle switch: DC
    \end{itemize}
  \item Oscilloscope:
    \begin{itemize}
      \item Operating mode: X-Y Mode
      \item Channel I: ≥10 mV/DIV (DC)
      \item Channel II: 0.5 V/DIV. (DC)
    \end{itemize}
  \item Function generator:
    \begin{itemize}
      \item Function: ~ (Sine)
      \item Amplitude: Middle position
      \item Attenuation: 20 dB
      \item DC -offset: 0 V (DC button pressed)
      \item Sweep button: Pressed
      \item Mode*: 'C u
      \item Stop*: 8.5 MHz
      \item Start*: 7.5 MHz
      \item Period*: 100 ms approx. (fast sweep)
    \end{itemize}
\end{itemize}

* Press button and set desired value with knob

Having found the resonance value, bring the start and stop values nearer together until the signal is clearly visible, e.g. between 8.3 and 8.5 MHz.
Measuring

Preparation

Finding the signal:
- Operate the oscilloscope in XY-mode.
- Switch off the oscilloscope’s storage mode.
- If necessary, switch off the sensitivity of oscilloscope channel I.
- Set the toggle switch of the I/V converter to DC.
- Set the start and stop frequencies on the function generator \( f_A = 8.3 \text{ MHz}, \ f_E = 8.5 \text{ MHz} \).
- Switch the function generator to a period of 100 ms (fast sweep).
- Start the function generator by pressing the button labelled MANUAL.
- Set the Helmholtz coil current \( I \approx 0.74 \text{ A} \) and vary it until an absorption signal can be seen on the oscilloscope screen.

Oscilloscope storage mode:
- Switch on the storage mode of the oscilloscope.
- Press the START button on the function generator.
- Set the horizontal deflection of the oscilloscope to \( x_A = 1.0 \) divisions.
- Press the STOP button of the function generator.
- Set the horizontal deflection of the oscilloscope to \( x_E = 9.0 \) divisions.

Fine adjustment:
- Start the function generator by pressing the button labelled MANUAL.
- Switch the function generator to a period of 10 ms (slow sweep).
- Switch the vertical deflection of the oscilloscope to sensitive.
- Turn the quarter wavelength plate back and forth between +45° and −45° and check whether all lines of the absorption spectrum appear on the oscilloscope screen.
- If necessary, readjust the Helmholtz coil current \( I \) or the start frequency \( f_A \) and stop frequency \( f_E \) accordingly.

Procedure

Note: with \( \sigma^+ \)-polarisation, the absorption line with the lowest frequency is the most intense.

For each of the spectra to be recorded:
- Set the quarter-wavelength plate to \( \sigma^+ \) polarisation.
- Start the function generator by pressing the button labelled MANUAL.
- Wait until the absorption spectrum has been completely recorded.
- Stop recording the absorption spectrum.
- Determine the amplitude \( V \) of the absorption lines.
- Check the start frequency \( f_A \) and the stop frequency \( f_E \).
- Check the current in the Helmholtz coils \( I \).

Settings

Oscilloscope:
- Operating mode: X-Y mode
- Storage mode
- Channel I: 10 mV/DIV (DC)
- Channel II: >0.5 V/DIV (DC)
- Recording range: 1.0 - 9.0 divisions
- Time base: 1 s/DIV

Function generator:
- Stop: 8.35 MHz
- Start: 8.45 MHz
- Period: 10 s (slow sweep)
- Amplitude: variable
- Attenuation: variable

Alternatively recording can be carried out using a CASSY module.

Now various spectra should be recorded in turn with different settings for the high-frequency amplitude and the preliminary attenuation.

For example:
- Attenuation 20 dB
- Amplitude: 1 / 2 / 3 / 4 / 6 / 8 divisions
- Then
- Attenuation: 0 dB
- Amplitude 0.5 / 1 / 1.5 / 2 / 2.5 / 3 / 4 / 5 divisions

Since 20 dB of attenuation results in a 10-fold reduction in amplitude, a setting of 1 division with attenuation of 20 dB can be calculated to correspond to 0.1 divisions with attenuation of 0 dB.
Measuring example

Fig. 5: Absorption spectrum for $^{87}$Rb with $\sigma^+$ light at 20 dB attenuation and high-frequency amplitude increasing from top to bottom.

Fig. 6: Absorption spectrum for $^{87}$Rb with $\sigma^+$ light at 0 dB attenuation and high-frequency amplitude increasing from top to bottom.
Evaluation

The rise of normal lines can be easily observed in Fig. 5 with low power and 20 dB attenuation, while the $\Delta m_F = \pm 2$ transitions can be observed in the last spectrum of Fig. 5 and in those of Fig. 6.

By way of example the height of the lines in each spectrum has been determined with respect to the lowest frequency lines. For the HF amplitude, the 20 dB attenuation in Fig. 5 has been calculated to correspond to a factor of 0.1 of a scale division.

Now, for example, the following values result for the lowest-frequency normal line and the lowest-frequency $\Delta m_F = \pm 2$ line:

<table>
<thead>
<tr>
<th>HF divisions</th>
<th>$\Delta m_F = \pm 1$ / V</th>
<th>$\Delta m_F = \pm 2$ / V</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.0168</td>
<td>0</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0342</td>
<td>0</td>
</tr>
<tr>
<td>0.3</td>
<td>0.044</td>
<td>0</td>
</tr>
<tr>
<td>0.4</td>
<td>0.0538</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0539</td>
<td>0</td>
</tr>
<tr>
<td>0.6</td>
<td>0.0647</td>
<td>0</td>
</tr>
<tr>
<td>0.8</td>
<td>0.0745</td>
<td>0.0092</td>
</tr>
<tr>
<td>1</td>
<td>0.0722</td>
<td>0.0122</td>
</tr>
<tr>
<td>1.5</td>
<td>0.071</td>
<td>0.0428</td>
</tr>
<tr>
<td>2</td>
<td>0.0759</td>
<td>0.0771</td>
</tr>
<tr>
<td>2.5</td>
<td>0.0747</td>
<td>0.1016</td>
</tr>
<tr>
<td>3</td>
<td>0.0808</td>
<td>0.0979</td>
</tr>
<tr>
<td>4</td>
<td>0.0845</td>
<td>0.1028</td>
</tr>
<tr>
<td>5</td>
<td>0.082</td>
<td>0.1077</td>
</tr>
</tbody>
</table>

Initially, at low power, the amplitude of the $\Delta m_F = \pm 1$ transitions starts to rise and a matching gradient indicates an exponent A close to 1, i.e. a linear dependency between the height of the lines and power.

In the middle of the plot, the amplitude of the $\Delta m_F = \pm 1$ transitions saturates due to the fact that participating levels are equally occupied. At the same power, the $\Delta m_F = \pm 2$ transitions start to increase in number.

A matching straight line then indicates a different power dependency with an exponent close to 2, i.e. a quadratic dependency on power, as would be expected for a two-quantum process.

Note

Looking more closely at the bottom two curves in Fig. 6, it is already possible to see $\Delta m_F = \pm 3$ transitions. The line at 8.393 MHz becomes much more apparent at this point.

Literature


If this is now plotted in Fig. 7, it is useful to use a double logarithmic scale, whereby polynomials appear as straight lines of corresponding gradient.

Fig. 7: Power dependency of transitions from Fig. 5 and Fig. 6