Determining Plank's constant
Counter voltage method

Objects of the experiment

■ Record the voltage-current characteristic of a photocell.

■ Measure the kinetic energy of the electrons as a function of the frequency of the light.

■ Determine Planck's constant h

Principles

Light interacting with matter can transfer energy to surface electrons and light of a sufficiently short wavelength can even liberate them from the surface in certain metals (photoelectric effect). The energy of the electrons depends on the frequency ν of the incident light, but not on the intensity. The intensity only determines the number of liberated electrons. This fact contradicts the principles of classical physics, and was first interpreted in 1905 by Albert Einstein. He postulated that light consists of a flux of particles, called photons, whose energy E is proportional to the frequency:

$$E = h \bullet \nu \qquad (I)$$

The proportionality factor h is known as Planck's constant, and is regarded as a constant of nature. In this particulate conception of light, each photoelectron is released by a photon and exits the metal surface with the kinetic energy

$$E_{kin} = h \bullet \nu - W_K \qquad (II),$$

where W_K is the work function of the irradiated material.

We can determine Planck's constant h by exposing a photocell to monochromatic light, i.e. light of a specific wavelength, and measuring the kinetic energy E_{kin} of the ejected electrons.

Fig 1: Schematic representation of an experiment for measuring Planck's constant h with the aid of the photoelectric effect. Monochromatic light (produced by wavelength filter F) falls on cathode K of a photocell. The photoelectrons excited here travel to anode A against the counter voltage Uc and create a photocurrent Ip.
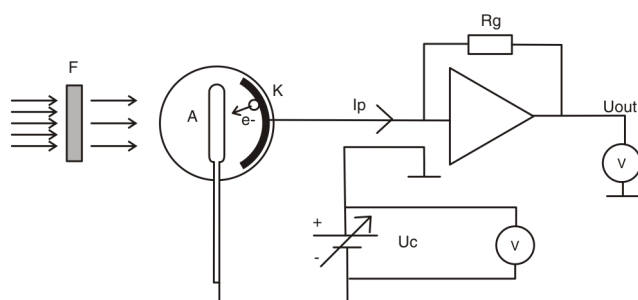

Fig. 1 shows a schematic representation of such an experiment. The light falls through an annular anode A, here a platinum wire, onto a potassium surface K. Thanks to its low work function—the valence electrons of alkali metals are weakly bound—potassium is a very suitable cathode material.

The photocathode K is held close to ground by the amplifier. There is a adjustable voltage Uc (the "counter voltage") applied between anode and ground and therefore between anode and cathode, usually with the anode at a more negative potential than the cathode. Some of the ejected photoelectrons travel to the anode, and if the kinetic energy of the electron is larger than the energy loss while "climbing up" the potential e $\bullet$ Uc, they reach the anode and are registered as a photoelectric current Ip.

The energy of an electron reaching the anode is:

$$E = h \bullet \nu - W - e \bullet U \qquad (III)$$

e: elementary charge

Here, W is not only the work function W_K of the cathode, as additionally the contact potential between the cathode and the anode is included in the energy balance.

If the remaining energy of an electron is still positive it can reach the anode and contribute to the photocurrent, if the calculated energy would be negative, the electron will not reach the anode wire, but turn away and form an electron cloud in front of the cathode.

The voltage at which the photoelectric current reaches zero is in some other experiments called the limit voltage U_0. In this experiment we record the total voltage-current dependency of the cell to make a better estimation of the limit voltages.

The current-voltage traces are recorded for various wavelengths λ and therefore frequencies

$$\nu = \frac{c}{\lambda} \qquad (IV)$$

c: speed of light in a vacuum

of the incident light. When the frequency of the incident light increases by $\Delta\nu$, the electron energy increases by $h \bullet \Delta\nu$. The limit voltage will then increase by ΔU_0 to compensate for the rise in energy. This increase is independent of the work

function as long as we use the same area of the photocathode.

When we plot the limit voltage $U_0(\nu)$ as a function of ν, equation (III) gives us a straight line with the slope:

$$\frac{\Delta U_0}{\Delta \nu} = \frac{h}{e} \qquad \text{(V)}$$

For a known elementary charge e, this gives us Planck's constant h.

In reality there are some more currents summing up to the measured photoelectric current: The true photoelectric current from the electrons released in the cathode is of course the main component, but there are also contributions from a so called dark current of ohmic behavior, and a very small

component of electrons traveling from the anode wire to the cathode, being released from the anode by stray light.

In total, these three currents sum up, so that at a sufficiently small negative potential there is a large current of electrons flowing from the illuminated cathode to the anode, which we call a positive current Ip. With increasing negative anode potential Uc, the photocurrent decreases exponentially and changes sign to a negative current, due to the leakage current effects discussed.

In this experiment, a direct view prism inside a compact arrangement is used to select the wavelengths. The spectral lines from the light of a high-pressure mercury lamp are separated by a prism and selected by a moveable slit inside the optical arrangement.
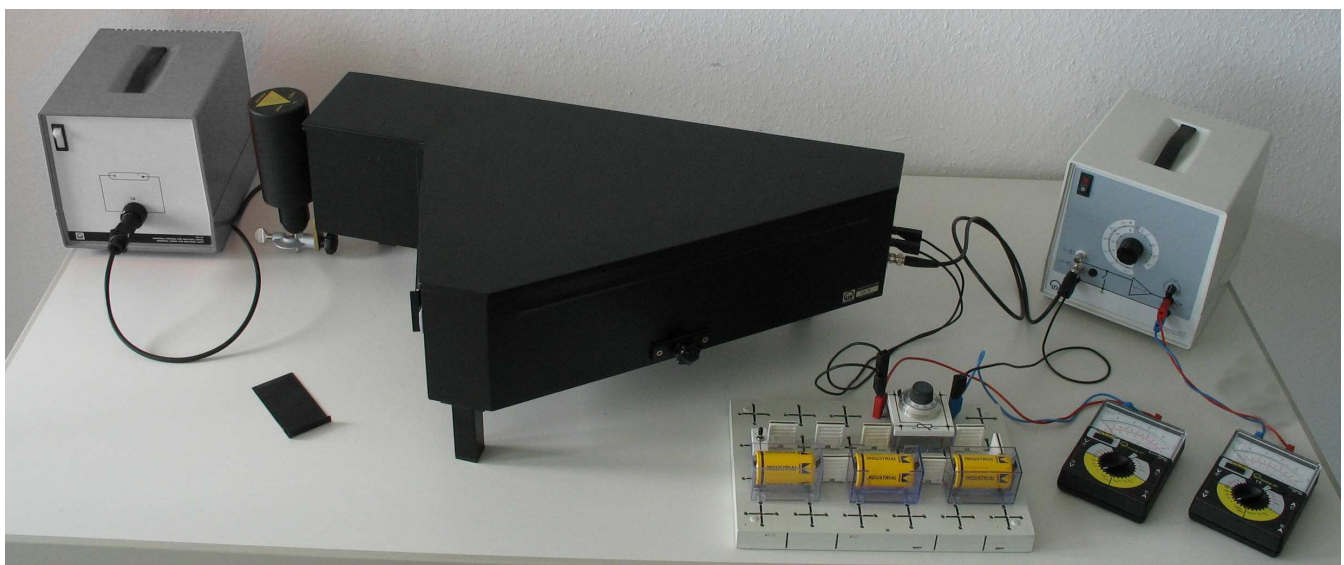

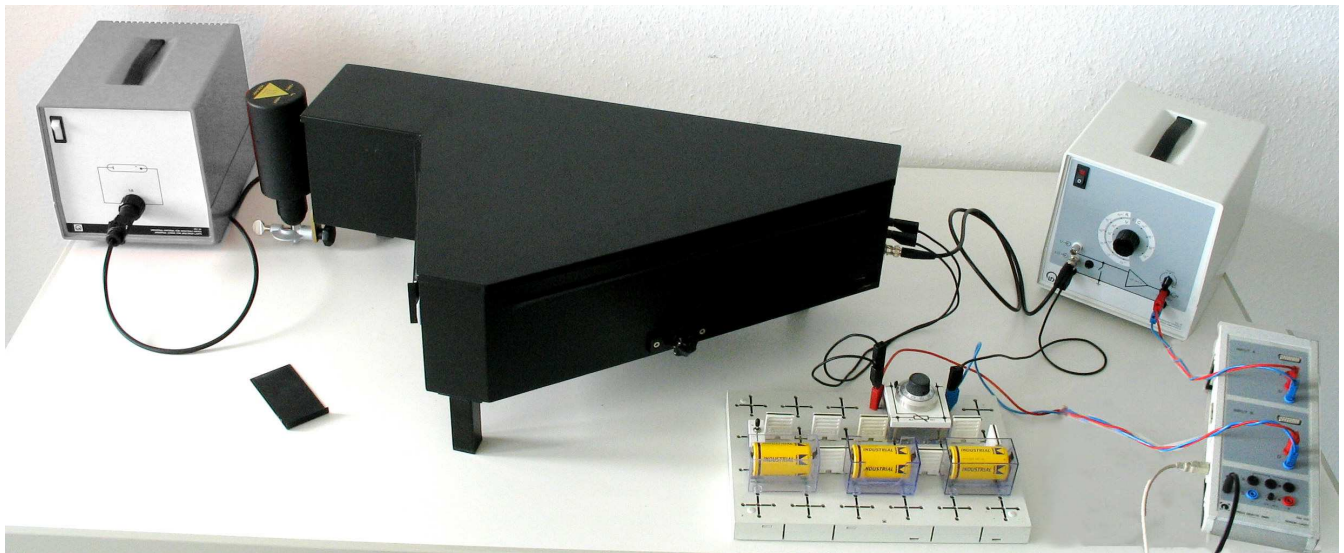Fig 2a: Setup of the experiment with multimeters

Fig 2b: Setup of the experiment with CASSY

Setup

Optical setup:

Note: The high-pressure mercury lamp reaches its full intensity after a ten-minute warm-up period. Switch on the high-pressure mercury lamp when you begin setting up the experiment, so that you can start measuring as soon as you are finished. Do not start to record values while the lamp heats up.
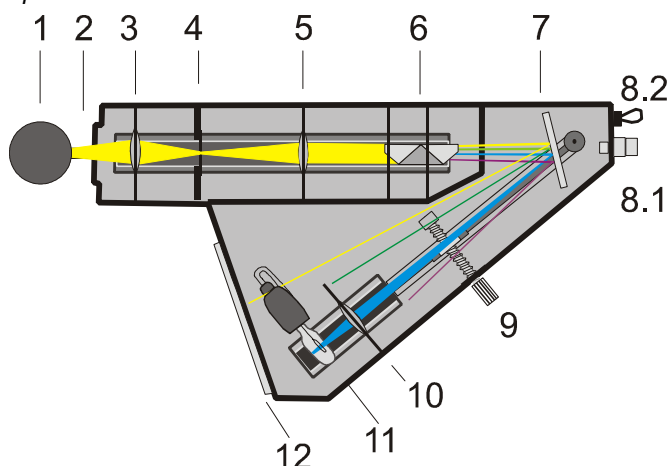

Fig 3 The optical path inside the compact arrangement.

Setting up the experiment:

- cover the inner side of the emission window (12) with white paper. Remove the direct-vision prism (6) and the lens with slit diaphragm (10) from the beam path. Make the necessary electrical connections as visible in Fig 2,

- Connect the universal choke to the mains.

- Secure the Hg lamp (1) at a distance of 5 mm in front of the compact arrangement housing and switch on;

- Form an image of the Hg lamp (1) on the slit (4) with the lens (3) (adjust the lamp and lens).

- Form an image of the slit on the emission window (12) with lens (5) by adjusting lens (5) and, if necessary, mirror (7).

- Insert the direct-vision prism (6) so that the Hg lines are on the inside at the emission window with the violet line on the outside (ensure that the black rubber collar is fitted in such a way that no stray light can pass through).

- Screw the photocell into the socket; and align the photocell so that the blue-black coated surface is facing towards the mirror.

- Place the lens with mounted slit diaphragm in the beam path so that the image of a spectral line is formed on the photo cathode.

- Adjust lens (5) so that a sharp image of the spectral lines is obtained on the slit (10). Due to chromatic error either the yellow/green lines are perfectly sharp or the blue ones, so find a setting in between.

The light from the mercury lamp should now produce a light spot on the coating (the sensitive area) of the photocell.

The light should not fall on the metal ring nor on the part of the black-coated area to which the contacts are attached. The edge zones should not be illuminated either.

Note: Take care that the photocell remains clean, especially free from dust and grease, as these can become conductive, clean the outside if necessary.

Apparatus

1 Photo cell for determining Planck's constant	558 77
1 Compact arrangement	558 79
1 High pressure mercury lamp	451 15
1 Universal choke, in housing, 230 V, 50 Hz	451 30
1 I amplifier D	532 00
2 or 3 Monocell holder	57686
2 or 3 Mono cell 1.5 V (IEC R20)	68548
1 10-turn potentiometer, 1 kOhm,	57793
1 Toggle switch, single-pole	57913
1 Plug-in board A4	57674
1 Set of 10 bridging plugs	50148
or alternatively	
1 Power supply 12 V DC	521 49
2 Voltmeter, DC	e.g. 531 100
or alternatively	
1 Sensor-CASSY	524010 S USB
1 PC	
Connecting leads	

Electrical assembly:
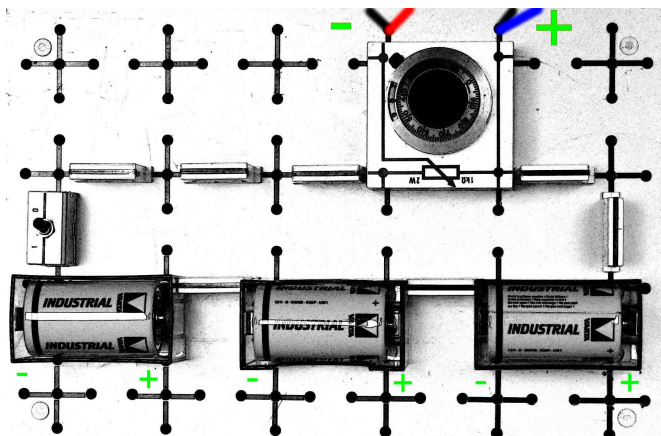
The photoelectrons incident on the metal of the photocell create a photocurrent which is amplified by the I measuring amplifier D and converted to a voltage. This voltage can then be read by a voltmeter or recorded by the computer, for example with a CASSY

Note: this experiment requires the use of a floating power supply, the plug-in board can be replaced by a power supply like 521 49, but any power supply with ground referenced outputs must not be used.

- Attach the BNC cable of the photocell inside the compact arrangement to the measuring Amplifier D.

- Set up the STE plug-in board (f) with the batteries and the potentiometer as shown in the photo above, usually two batteries in series will be enough to do the experiment.

- Connect both black 4mm cables of the photocell to the negative (-) output of the plug-in board, which is the wiper of the potentiometer.

- Connect the positive output (+) of the plug-in board to the ground connection of the measurement amplifier D, see Fig. 1 for the schematic diagram.

- Connect one multimeter or CASSY input A,U (not I) to the output of the measurement amplifier D and the other multimeter or CASSY Input B to the output of the plug-in board.

Carrying out the experiment

Notes:

If potassium from the light-sensitive layer of the cathode becomes deposited on the anode ring, this can cause an electron flux which will interfere with the experiment. If necessary, bake out the photocell as described in the Instruction Sheet.

Dirt on the photocell can cause leakage currents between the anode and the cathode which can affect the measurement of the limit voltage U_0. Clean the photocell with alcohol.

You do not need to darken the room; this has no effect on the measurement results.

- Switch on both multimeters and set the range switch to +-10 V DC or start CASSYLab on the computer connected to the CASSY, select the 10 V range for both inputs A and B.

- Tune the compact arrangement for yellow light (λ_{Hg} = 578 nm). Adjust the swiveling arm using the threaded guide (9) so that the shadow of the marking pin is on the yellow spectral line.

- Set the potentiometer for the maximum counter voltage, something like – 3.0 Volts.

- Observe the photocurrent, note that in the most sensitive setting of the I-Amplifier D the transimpedance Rg is 100 GΩ. So a photocurrent of 10 pA will be equivalent to 1V output. In CASSYLab create a formula for a Symbol "I", with Unit "pA" and the formula "UA1*10", meaning 1 V is worth 10 pA

- In CASSYLab, select "Measuring parameters / manual recording", and change the axis assignment to x-axis = "UB1", y-axis= "I"

- Reduce the counter voltage stepwise (for example in 0.5 V steps) and record the resulting photocurrent. When the photocurrent starts to rise, use smaller steps and wait before reading the values until the photocurrent has settled. In CASSYLab use "F9" to record one value.

- Tune the compact arrangement to green light (λ_{Hg} = 546 nm) and repeat the measurement. In CASSYLab select "Measuring parameters / Append new meas. Series" to start a new curve in a different color.

- Repeat the measurement with the blue (λ_{Hg} = 436 nm), and violet (λ_{Hg} = 405 nm) lines.

If there is no photocurrent at all observable, check if the photocell is fully screwed into the socket.

Measuring example
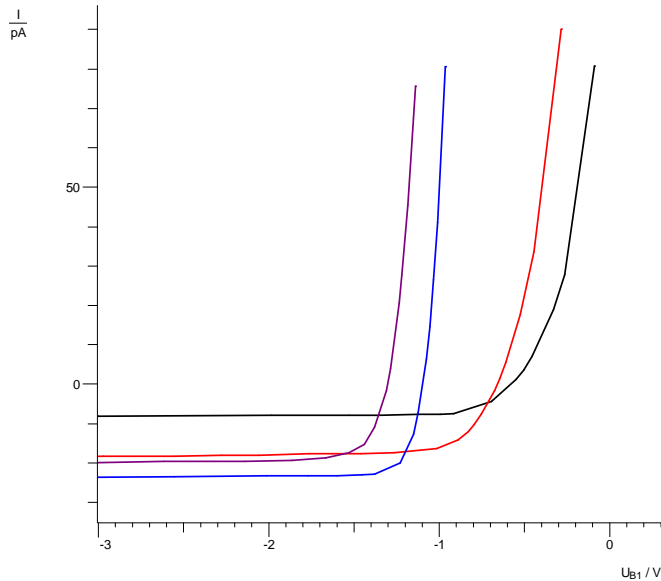
Below a sufficiently negative counter voltage, all the recorded curves show a constant current, which is independent of the counter voltage but depends on the intensity and wavelength of the incident light. This reverse current is generated by stray light on the anode ring. There is no ohmic component in this example, so the cell was clean of dust and dirt on the outside.

Depending on the wavelength of the incident light, the photo-current starts to rise at different counter voltages. For the violet line, this happens at –1.5 Volt, for the yellow one at –0.6 Volt, and the other wavelengths in between.

Evaluation

There are several ways in the literature to evaluate the measured data and determine at what counter voltage exactly the photocurrent starts.
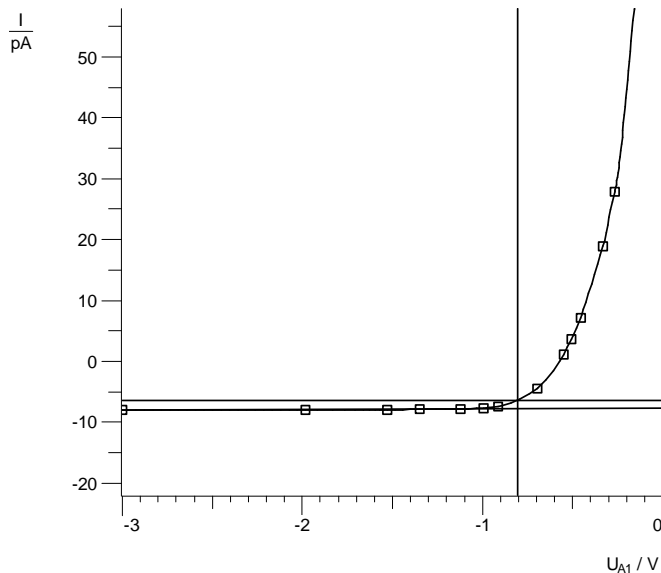

The easiest approach is to draw a line through all the measurement points on the left side before the photocurrent start, then draw a parallel line a few pA apart and take the crossing of the photocurrent with this line, in the image above at –0.81 V.

Another approach is to take again a best fit line through all the points on the left, and then another best fit line through all the data points at high photocurrents and look for the intersection of both lines, here at –0.4 V.
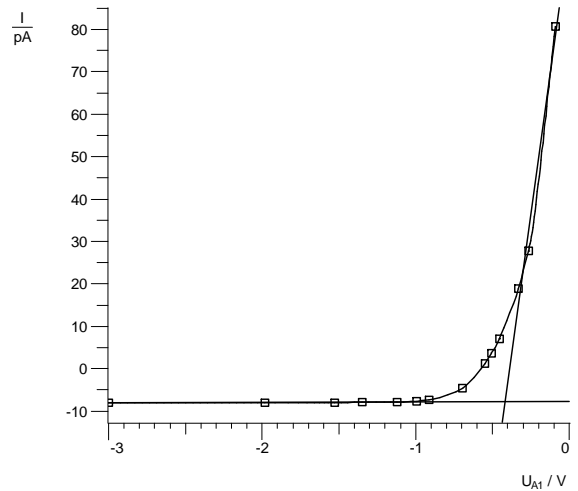
A third approach, which requires much more measurements, would measure the current-voltage curves for the same wavelength but at different intensities, and take the point where the curves intersect as "the" counter voltage, see [1]

A fourth, purely mathematical approach fits an exponential function to the measured values.

A = 8,1999, B = 4,9749, C = -0,5686 (A*(exp(B*(x-C))-1))
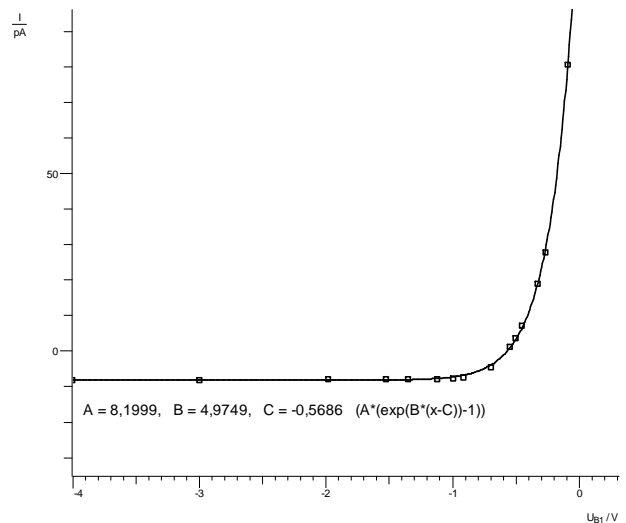
A function similar to Shockley's Diode equation $I(U) = A \cdot \left(e^{B \cdot (U - C)} - 1 \right)$ is used, where A is the dark current, B somewhat related to light intensity times photocathode response and C is the counter voltage. In CASSYLab, press the right mouse key in the diagram, select "Fit function", "Free fit"; in the dialog box type the formula "A*(exp(B*(x-C))-1)", make some rough estimations for A(=10), B(=7) and C(=-1) and mark the data points to start the free fit. A few seconds later, CASSYLab will display the optimum values for A, B and C, and C is our result, where $I(C) = 0$. Therefore this evaluation is very similar to P6.1.4.1, in that experiment we are simply measuring the voltage where $I = 0$

In the end, all the different evaluations give quite similar results for Planck's constant as it is calculated only from the slope of the threshold voltage versus photon energy. Similar as the work function cancels out, so does any offset error from the data evaluation.

For simplicity we have chosen to use the first method described and look at the point where the photocurrent starts to

barely rise above the baseline on the left side of the diagram from the other currents.

This gives us the values of the limit voltage versus frequency of light:

λ / nm	ν / THz	U_0 / V
578	519	0,95
546	549	1,2
436	688	1,5
405	741	1,8

Plotting the data in a diagram gives us this result
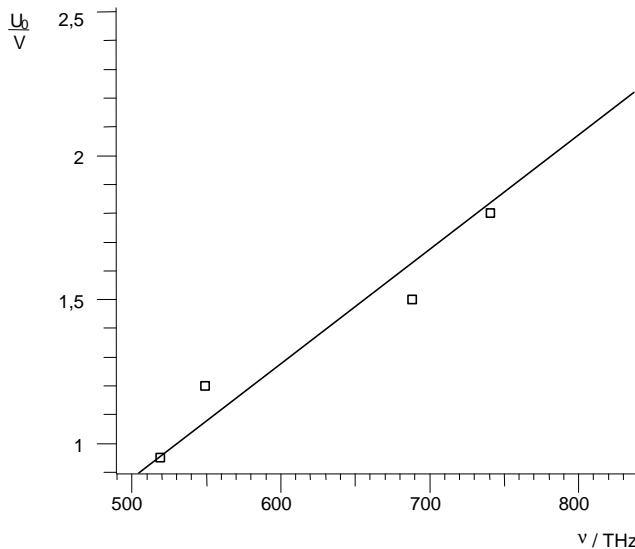
The plotted measurement points lie on a straight line with close approximation.

A line fitted to the measurement points has a slope of

$$\frac{\Delta U_0}{\Delta \nu} = 0,00398 \text{V/THz} = 3.98 * 10^{-15} Vs$$

According to (V), multiplying with e = 1.6 • 10^{-19} As gives a value of Planck's constant

h = 6.37 • 10^{-34} Js

Literature value: h = 6.62 • 10^{-34} Js

Results

In the photoelectric effect, the kinetic energy E_{kin} of the liberated electrons depends on the frequency, and not on the intensity of the incident light.

Planck's constant h can be determined by measuring the limit voltage U_0, above which the electrons can no longer escape, as a function of the frequency ν.

Acknowledgment

Special thanks to Dr John Fry, Department of Physics, University of Liverpool for very helpful discussions and the idea of fitting with an exponential function.

References

[1] Langensiepen, Fritz; „Ein verbessertes Verfahren zur Bestimmung des Planckschen Wirkungsquantums mit in der Schule vorhandenen Mitteln."; Praxis der Naturwissenschaften. Physik, 30 (1981) 11, S. 321-328