Determining Plank's constant
Measurement with a compact arrangement

Objects of the experiment

■ Measure the kinetic energy of the electrons as function of the frequency of light

■ Determine Planck's constant h

Introduction

The Planck's constant

$$h \quad \text{or} \quad \hbar = \frac{h}{2\pi}$$

is a one of the fundamental constants of physics which occurs in describing the quantum character of microphysical objects, as for example, electrons and photons. The Planck's constant, therefore, occurs as fundamental constant in many experiments describing quantum behavior of nature.

In this experiment the Planck's constant h will be determined by means of the photoelectric effect.

Principles

Electrons may be released from a metal surface due to the irradiation with light (photoelectric effect). The number of "photo electrons" depends on the intensity of light. The energy of the released electrons, however, depends only on the frequency of the light. This experimental findings was explained by Einstein in 1905 by assuming that light is composed of a stream of particles − the so-called "photons" − and it is assumed that each photoelectron is released by an individual photon, whose energy is proportional to the frequency:

$$E = h \cdot \nu \tag{I}$$

The "Einstein relationship" describes the conservation of energy for these process. Each released electron takes up the energy $h \cdot \nu$ of a photon. The amount of energy exceeding the work function W is retained by the electron as kinetic energy E_{kin}:

$$E_{kin} = \frac{1}{2} m v^2 = h \cdot \nu - W \tag{II}$$

The work function of the electron depends on the material.

The Planck's constant h can be determined by exposing a photocell to monochromatic light (i.e. light of a specific wavelength) and measuring the kinetic energy E_{kin} of the released electrons.

Fig. 1 shows a schematic representation of such an experiment. Light falls through a ring-shaped anode (platinum wire) onto a potassium surface. The photoelectrons reach the anode and are measured in form of a photocurrent I.

If the photoelectrons are ejected against a negative potential which is gradually increased, the photo-current continuously decreases. The

voltage at which the photo-current reaches precisely zero is called the limit voltage U_0. At this voltage even the weakest bounded electrons, i.e. those with the lowest work function W and thus with the greatest kinetic energy E_{kin}, can no longer overcome the anode voltage.

In this experiment the anode voltage is generated using a capacitor which is charged by the incident electrons up to the limit voltage U_0. The limit voltage U_0 can be used to calculate the kinetic energy of these weakly bound electrons:

$$e \cdot U_0 = h \cdot \nu - W \tag{III}$$

e: elementary charge

W: work function (see supplementary information)

If the frequency of the incident light is increased by $\Delta\nu$ the energy of electron increases by $h \cdot \Delta\nu$.
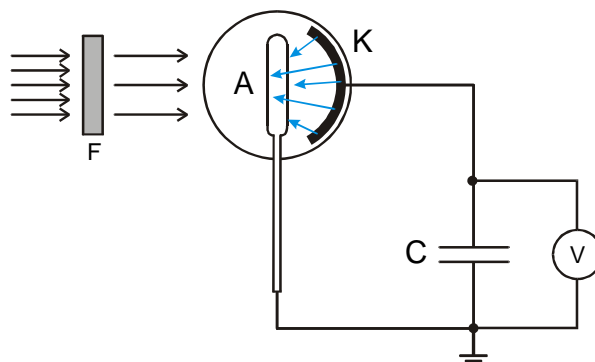

Fig. 1: Schematic representation of the setup for measuring Planck's constant with the aid of the photoelectric effect. Monochromatic light is produced by a wavelength filter F falls on the cathode K of a photocell. The photoelectrons travel to the anode and charge a capacitor C up to the limit voltage U_0.

The limit voltage must be increased by ΔU_0 to compensate for the photocurrent. For this situation the following equation applies:

$$e \cdot \Delta U_0 = h \cdot \Delta \nu \qquad \text{(IV)}$$

i.e. the energy increase $h \cdot \Delta \nu$ is just compensated by the energy loss $e \cdot U_0$.

If limited voltage U_0 is plotted as function of ν equation (IV) gives a straight ling with the slope:

$$\frac{\Delta U_0}{\Delta \nu} = \frac{h}{e} \qquad \text{(V)}$$

For a known elementary charge e the Planck constant h can be determined from the slope of the plot $U_0(\nu)$.

Apparatus

|	
1 Photocell for determining h	558 77
1 Compact arrangement for h	558 79
1 High pressure mercury lamp	451 15
1 Universal choke	451 30
1 Electrometer amplifier	532 14
1 Plug-in power supply 12 V AC	562 791
1 Capacitor 100 pF, 630 V	578 22
1 Key switch (NO), single-pole	579 10
2 Clamping plug	590 011
1 Multimeter LD-analog 20	531 120
1 Screened cable BNC/4 mm	575 24
1 Distribution box	502 04
1 Pair cables, 50 cm, red/blue	501 45
1 Pair cables, 100 cm, black	501 461
1 Connecting Lead, 25 cm, black	500 414
1 Connecting Lead, 100 cm, yellow/green	500 440
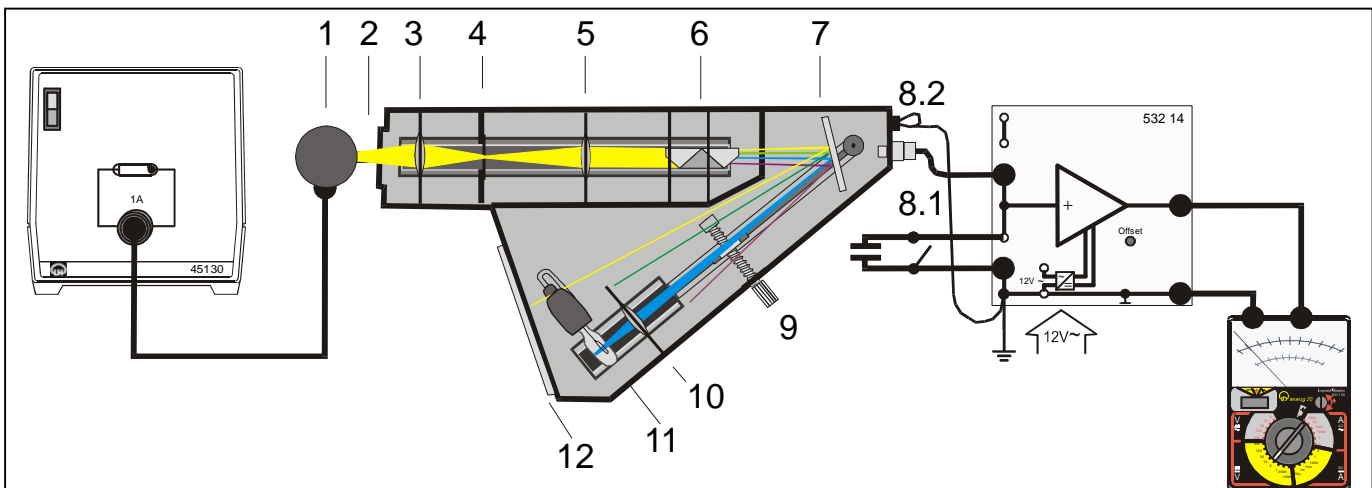
Fig. 2: Experimental setup schematically.

(1)	high pressure mercury lamp	(2)	slider
(3)	converging lens	(4)	slit
(5)	imaging lens	(6)	direct vision prism
(7)	mirror		
(8.1)	socket for screened cable BNC/4 mm (cathode)		
(8.2)	4 mm sockets (ring anode)	(9)	swiveling arm
(10)	converging lens with slit diaphragm	(11)	photocell
(12)	window and dimming slider		

Setup (electrical assembly)

The photoelectrons incident on the metal ring anode of the photocell charge a capacitor, and thus, generating the limit voltage U_0 required for determining the kinetic energy. The electrometer amplifier is used to measure the voltage at the capacitor.

Set up the electrometer amplifier circuit as shown in Fig. 2. and Fig. 3.

- Attach the clamping plugs (f) and connect the 100 pF capacitor and the key switch (Fig. 3).
- Attach the screened cable BNC/4 mm to BNC socket (8.1 in Fig. 2) and to the electrometer amplifier (g); connect the ground connection of the screened cable BNC/4mm to the ground on the electrometer amplifier (h in Fig. 3).
- Connect both 4 mm sockets (8.2 − cable loop in Fig. 2) by the cable 25 cm. Connect a 4 mm socket by a cable 100 cm to the ground connection on the electrometer amplifier (h in Fig. 3).
- Connect the multimeter to the output of the electrometer amplifier by the cables 50 cm (Fig. 2).
- Connect the ground of the electrometer amplifier with the ground connection of the distribution box by using the cable yellow/green.
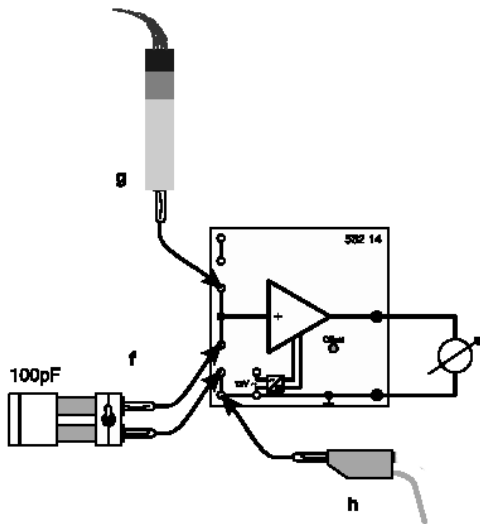
Fig. 3: Electrometer amplifier circuit for measuring the limit voltage U_0.

Setup (Optical adjustment)

Preparation according Fig. 2: cover the inner side of the plexiglass plate of the emission window (12) with a sheet of white paper. Remove the direct vision prism (6) and the lens with the slit diaphragm (10) from the beam path. Make the electrical connections as described under "setup electrical connections" (see also Fig. 3). Secure the high pressure mercury lamp (1) at a distance of about 5 mm in front of the housing of the compact apparatus and switch on.

- Form an image of the high pressure mercury lamp (1) on the slit (4) with the lens (3). Adjust the lamp and lens (slit in the center of the holder).
- Form an image of the slit on the emission window with the lens (5) by adjusting lens (5) and, if necessary, mirror (7).
- Insert the direct-vision prism (6) so that the both the violet line and the yellow line of the high pressure mercury lamp strikes the emission window.
- Screw photocell into the socket; place the lens with mounted slit diaphragm in the beam path so that an image of a spectral line is formed on the photocathode.
- If necessary, repeat the second step.

Hint: Keep the optical surfaces clean. The result of dust and fingerprints is stray light leading to an excessively high limit voltage U_0. at low frequencies.

Carrying out the experiment

Notes:

The voltage of the capacitor can be influenced by induction effects. Move this part as little as possible during the experiment

Dirt on the photocell can cause leakage currents between the anode and cathode which can effect the measurement of the limit voltage U_0. Clean the photocell with alcohol. For further information see instruction sheet 558 77 of the photocell.

- Partially darken the room. The image of the slit diaphragm of the lens (10) is visible on the emission window.
- Switch on the multimeter and set the range to 3 V DC.
- Determine the limit voltage U_0 of the yellow light: To do this, adjust the swiveling arm using the thread guide (9) so that the shadow of the yellow line is visible at the emission window.
- Cover the emission window (12) with the dimming slider if you the experiment is not performed in a dark room.
- Discharge the capacitor by holding the down the key switch until the multimeter is zero.
- Start the measurement by releasing the key switch; wait about 30 s to 1 minute, until the capacitor has charged to the limit voltage U_0.
- Write down the measured value for U_0.
- Repeat these steps for green, blue and violet spectral lines.

Measuring example

Table 1:   Measured limit voltage U_0 as function of the wavelength λ and frequency ν.

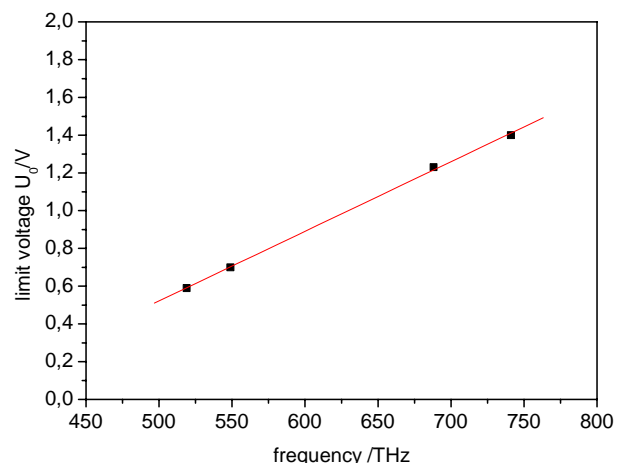| color	$\dfrac{\lambda}{nm}$	$\dfrac{\nu}{THz}$	$\dfrac{U_0}{V}$
yellow	578	519	0.59
green	546	549	0.70
blue	436	688	1.23
violet	405	741	1.40

Fig. 4:   The limit voltage U_0 as function of frequency ν (see Table 1).

Note: The frequency of the various lines of the high pressure mercury lamp can be determined using a prism or grating spectrometer, like e.g. described in experiment P5.7.1.1 and P5.7.2.1.

Evaluation and results

The limit voltage U_0 is plotted against the frequency ν of the high pressure mercury spectral lines in Fig. 4. The plotted measurement points lie on a straight line. From the fit of a straight line the slope is determined to:

$$\frac{\Delta U_0}{\Delta \nu} = 0.38 \cdot 10^{-14} \, \text{Vs}$$

With the elementary charge

$$e = 1.6 \cdot 10^{-19} \, \text{C}$$

it follows for the Planck's constant:

$$h = 6.1 \cdot 10^{-34} \, \text{Js}$$

Literature value: $h = 6.62 \cdot 10^{-34} \, \text{Js}$

The kinetic energy of the liberated photoelectrons depends on the frequency. Planck's constant can thus be determined by measuring the limit voltage U_0 for various frequencies above which the electrons can no longer escape.

Supplementary Information

1. The work function depends on the material. The use of potassium as cathode material is due to the particularly low working function of alkaline metals in which the electrons are weakly bound.

2. In order to obtain a deep understanding of the processes involved in the photoelectric effect it is necessary to study the energy distribution of the electrons in metals. From the plot of the kinetic energy $e \cdot U_0$ as function of the frequency of the irradiated light the work function can be determined in accordance with equation (II) from the ordinate intercept.

   However, this is not the work function of the cathode, as one might initially suppose. It is principally impossible to measure the work function of the cathode:

   Electrons in the cathode are in a potential well with a depth W_C which is measured from the Fermi-level E_E. In the same way, the electrons from the anode are also in a potential well with a depth of W_A.

   If a electrically conducting connection is created between the anode and cathode, the Fermi-levels adjust so that they are at the same height (Fig. 5).

   If a voltage U is applied between the anode and cathode the Fermi-levels are displaced by $e \cdot U$ with respect to each other. Electrons from the cathode must now overcome the highest point of the potential barrier which is given by:

   $$W_A + e \cdot U_0 \tag{VI}$$

   The necessary energy to overcome the potential barrier is supplied by the photon. The anode current becomes zero when

   $$h \cdot \nu = W_A + e \cdot U_0 \tag{VII}$$

   However, W_A cannot be determined easily due to their dependence on the crystal orientation and surface accumulation of gas atoms.

3. Historically, the quantum behavior of light which was suggested by equation (II) was being debated until the Compton's experiment confirmed independently the particle behavior of light. However, in contrast to photoelectric effect the Compton effect (scattering of photons on weakly bound electrons, see e.g. experiments P6.3.7.2 or P6.5.6.1) − where only a part of the photon energy is transferred to an electron − the photon energy is completely absorbed in the photoelectric process.
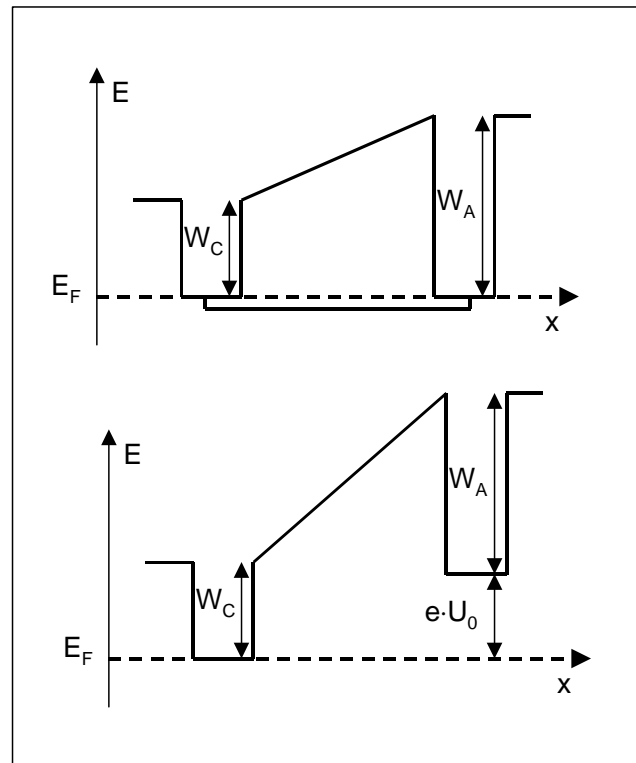

Fig. 5: Energy conditions for two metals (cathode and anode) with different work functions.