

Laser Doppler anemometry with CASSY

Objects of the experiment

- To detect interference at the intersection of two coherent rays
- To measure the frequency shift for two light rays which are being scattered by moving particles
- To determine the velocity of particles moved by a liquid
- To compare flow rates at the centre and close to the edge of a tube
- To compare the results with the medial flow rate of the liquid

Principles

Laser Doppler anemometry (LDA for short) is a contact-free, optical measuring method for determining the flow rate of a liquid or gas by determining the velocity of small particles in that flow. If the particles flow through the measuring volume of the LDA they scatter light whose frequency is shifted on account of the Doppler effect. The magnitude of the frequency shift is determined and this is used to calculate the particle velocity and therefore the flow rate.

There are two models for describing laser Doppler anemometry:

a) In the first case the Doppler shift is considered as is experienced by the light from the small particles moving at a velocity \vec{v} . For this the particle is first considered as a moving receiver which is illuminated by a stationary source. For the Doppler shift in this case only the speed component in the direction of the light spread \vec{l} makes a contribution (see figure 2). This leads to a first component of the complete Doppler shift of

$$v = v_0 \left(1 - \frac{\vec{v} \cdot \vec{l}}{c \cdot l} \right) \quad \text{for } (v \ll c). \quad (I)$$

For the scattered light, the particle then represents a moving emitter and the photo detector a stationary receiver. This leads to a further factor in the Doppler shift. Only the velocity component in the direction that the scattered light spreads \vec{k} contributes to the Doppler shift of this velocity component.

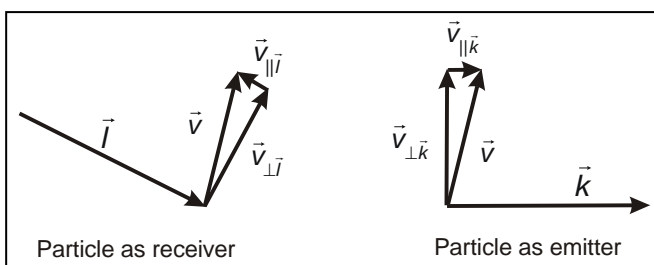


Fig. 1: Diagram showing light scattering by small moving particles



Fig. 2: Experimental setup

$$v = v_0 \frac{1}{1 - \frac{\vec{v} \cdot \vec{k}}{c \cdot k}} \quad \text{for } (v \ll c). \quad (II)$$

The frequency measured at the photo detector determined by both processes is:

$$v = v_0 \frac{1 - \frac{\vec{v} \cdot \vec{l}}{c \cdot l}}{1 - \frac{\vec{v} \cdot \vec{k}}{c \cdot k}}. \quad (III)$$

In this application the observed frequency shift is very small and is therefore difficult to observe. For this reason methods are used which avoid direct optical frequency measurements.

In the setup for this experiment the laser beam is split into two partial beams of equal intensity which are then superimposed inside the measuring volume (see figure 3). Particles passing through this zone scatter the light from both beams.

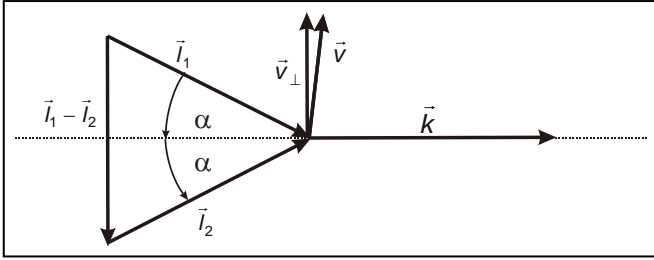


Fig. 3: Scattering with two superimposed light beams

The Doppler shift is in this case different for the two beams because they enter from different directions, but the scattered light is observed from the same direction (different vectors \vec{l} , identical vector \vec{k} , see figure 3).

The difference in the two frequencies, which is generally known as the interference frequency, is in this case called the Doppler frequency and is much smaller than the frequency of the light source, and furthermore it has a much smaller bandwidth. For this reason it is relatively easy to measure it electronically.

The dependency of the Doppler frequency on the velocity of the particles is determined as follows. According to equation (III) the frequency of the scattered light of the two partial beams is given by

$$v_1 = v_0 \frac{1 - \frac{\vec{v} \cdot \vec{l}_1}{c \cdot l_1}}{1 - \frac{\vec{v} \cdot \vec{k}}{c \cdot k}} \quad \text{and} \quad v_2 = v_0 \frac{1 - \frac{\vec{v} \cdot \vec{l}_2}{c \cdot l_2}}{1 + \frac{\vec{v} \cdot \vec{k}}{c \cdot k}} \quad (IV)$$

The difference is then

$$v_D = v_1 - v_2 = \frac{v_0}{c \cdot |\vec{l}_1 - \vec{l}_2|} \frac{\vec{v} \cdot (\vec{l}_1 - \vec{l}_2)}{1 - \frac{\vec{v} \cdot \vec{k}}{c \cdot k}} \quad (V)$$

The vector $\vec{l}_1 - \vec{l}_2$ is perpendicular to the median line between the two beam directions (see figure 3); for this reason in this experiment only the speed component v_{\perp} can be determined along this vector. Therefore, in our experiment the glass tube containing the flowing liquid is oriented parallel to this direction. Then, by measuring v_{\perp} the flow rate v is detected directly.

If only the scattering in the direction of the median line between the two beams is considered and only the velocity component v_{\perp} , the denominator is $1 - \frac{\vec{v} \cdot \vec{k}}{c \cdot k} = 1$. If, in addition, the angle between the two beams is 2α , one obtains:

$$v_D = v_1 - v_2 = \frac{v_0}{c} v_{\perp} \cdot 2 \sin \alpha = \frac{v_{\perp} \cdot 2 \sin \alpha}{\lambda} \quad (VI)$$

In the measurement v_D is determined, the wavelength of the laser $\lambda = \frac{v_0}{c}$ is known and the angle between the two beams 2α is determined by the geometric dimensions of the setup. This allows us to determine the velocity component v_{\perp} of the particle by measuring the Doppler frequency v_D .

$$v_{\perp} = \frac{v_D \cdot \lambda}{2 \sin \alpha} \quad (VII)$$

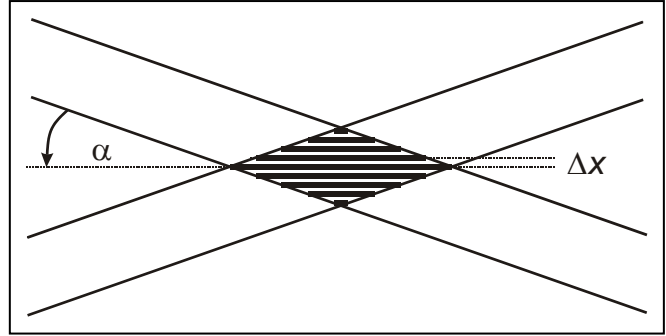


Fig. 4: Interference pattern in the superposition zone of two beams

b) The LDA principle can also be considered as an interference phenomenon. In the zone where the two laser beams cross, an interference pattern forms. If the superposition zone coincides with the foci of the beams, the wave fronts are straight and the zones of constructive and destructive interference are parallel and have a spacing Δx (see figure 4). This spacing depends only on the wavelength λ of the light used and the angle 2α between the two beams:

$$\Delta x = \frac{\lambda}{2 \sin \alpha} \quad (VIII)$$

If a particle moves through this interference pattern, it scatters light in the light areas but not in the dark ones. At the detector therefore a temporal variation in the intensity is measured; the temporal spacing Δt and therefore the frequency v_D of the variation depends on the distance between the interference strips Δx and the velocity component v_{\perp} of the particle perpendicular to it:

$$\frac{1}{\Delta t} = v_D = \frac{v_{\perp}}{\Delta x} = \frac{v_{\perp} \cdot 2 \sin \alpha}{\lambda} \quad (IX)$$

This equation is identical to equation (VI). Thus when considering it both ways, the same result is obtained for the dependence of the Doppler frequency on the velocity of the moving particles.

Summary

With both ways of considering the phenomenon therefore, the temporal variation of the signal at the photodiode depends only on the wavelength of the laser used, on the angle 2α between the two beams and on the velocity component v_{\perp} of the particle perpendicular to the median of the two beams. If, therefore, the frequency v_D of the variation is measured, the velocity component of the particles v_{\perp} can be determined.

Apparatus

1 optical bench, standard cross-section, 1 m ..	460 32
1 optical bench, standard cross-section 0.5 m	460 335
10 optics riders 90/50	460 374
1 He-Ne laser head, 5 mW	471 821
1 power supply for He-Ne laser, 5 mW.....	471 825
1 laser holder for laser 5 mW	470 010
1 holder for beam divider	473 431
1 beam divider 50%.....	473 432
1 cantilever arm.....	460 380
1 planar mirror with fine adjustment	473 461
1 extension rod	460 385
1 lens in frame, f = + 50 mm.....	460 02
1 lens in frame, f = + 100 mm.....	460 03
1 iris diaphragm.....	460 26
2 holders with spring clips	460 22
1 diaphragm with 3 diffraction holes	469 96
1 ring diaphragm from set of 4 different diaphragms	461 63
1 translucent screen	441 53
1 silicon photodetector	558 835
1 AC/DC amplifier, 30 W	522 61
1 screened cable BNC / 4 mm plug.....	575 24
1 pair of cables 50 cm, red/blue	501 45
1 connecting lead, 10 cm, red	500 401
1 set of 6 two-way plug adapters, red.....	501 641
1 resistor, 100 k Ω STE 2/19	577 68
1 separation funnel, 500 ml	602 404
2 silicone tubing, 7 mm \varnothing	604 433
1 tubing clamp after Hofmann, 20 mm	667 175
1 reaction tube, 200 x 8 mm \varnothing	664 146
1 holder for plug-in elements.....	460 21
2 small clip plug.....	590 02
1 stand base, V-shape, 28 cm.....	300 01
1 stand rod, 100 cm.....	300 44
1 stand ring with clamp, 100 mm \varnothing	666 546
1 micro spatula, 150 mm	604 5672
1 beaker, 150 ml, tall form.....	602 010
1 measuring beaker, clear SAN 500 ml.....	604 215
1 tape measure, 1 m / 1 mm	311 78
1 reflecting particles, 10 g	683 70
1 Sensor-CASSY.....	524 010USB
1 CASSY Lab	524 200
1 PC with Windows 95/98/NT or higher version	

Safety notes

The He-Ne laser 5 mW conforms to class 3B. Class 3B lasers are potentially dangerous if a direct or reflected beam impinges on the unprotected eye (direct view into the beam). As long as the appropriate information in the operating instructions is observed, experimentation with the He-Ne laser is not dangerous.

- Do not look into the direct or reflected laser beam!
- Wear suitable laser protection goggles.
- Avoid unintended mirror reflections (e.g. from watches, jewellery, tools with metal surfaces etc.)!
- All laser beams should be blocked at the end of the path by diffusely scattering material set up for this purpose.
- Before introducing new optical components into the setup (mirrors, beam splitters, lenses etc.) cover the laser beam or switch it off!
- View diffuse reflexes at a distance of at least 15 cm from the reflecting surface!

Setup and method

- Connect the Sensor-CASSY to the computer and start the measuring program CASSY Lab.
- Load the example.

Setup of the optical components

1. Align the optical benches and the auxiliary rail in parallel to the table and to each other:
 - Place the optical benches directly behind one another.
 - Adjust the height of the shorter optical bench to that of the other optical bench.
 - Rotate the shorter bench by 180° and set it up with its other end behind the other optical bench. Again adjust the height to match that of the optical bench.
 - Set up the longer optical bench with the other end behind the shorter optical bench and adjust the **height of the longer optical bench** to that of the shorter bench.
 - The optical should now be the same height at both ends.
2. Adjust the laser beam to be parallel to the shorter optical bench:
 - Attach the 5 mW He-Ne laser to the laser holder and attach it with two optics riders to one end of the shorter optical bench.
 - Adjust the laser beam to be parallel to the optical bench:
 - a) Attach the iris aperture on the optics rider to the shorter optical bench directly upstream of the laser and adjust to the desired height of the beam in the experimental setup. Close the aperture as much as possible. Slightly loosen the screws of the laser holder on the side of the aperture nearer to the laser. Using the lower two screws align the laser in such a way that the laser beam passes through the centre. Then carefully tighten the upper screw.

b) Attach the iris aperture to the shorter optical bench as far away from the laser as possible without changing its height. Slightly loosen the screws of the laser holder on the side of the aperture away from the laser. Using the lower two screws align the laser in such a way that the laser beam passes through the centre of the aperture. Then carefully tighten the upper screw.

- Repeat actions a and b until the beam passes through the centre of the aperture in both positions. Then remove the aperture.

Note:

By introducing a new optical element into the path of the beam, uncontrolled reflections can occur which may be dangerous for the observer:

- Cover the laser beam upstream of the new element or switch off laser.
- Cover the path of the beam directly behind the new element by means of a translucent screen.

Only uncover the path of the beam after the new element has been introduced and fixed, and then change the position of the translucent screen.

3. Adjust the plane mirror in such a way that the laser beam is deflected at right angles to the shorter optical bench.

- Attach the plane mirror in the extension rod to an optics rider several centimetres from the end of the shorter bench so that the beam impinges on the plane mirror at its centre and that the reflection returns in the direction of the laser. Adjust the vertical deflection of the mirror with the upper thumb screw so that the beam is reflected back at the same height.

- Set up the longer optical bench at approximately right-angles to the shorter bench upstream of the plane mirror. Attach the translucent screen to the end of the longer optical bench.

- Rotate the plane mirror by 45° so that the beam after reflection runs in parallel to the longer optical bench. In this way the plane mirror is no longer impinged on at its centre by the laser beam. Set up the iris aperture on the optical bench close to the plane mirror and close it as much as possible.

4. Adjust the laser beam to be parallel to the optical bench:

a) Shift the plane mirror on the auxiliary bench in such a way that the reflected beam just passes through the aperture.

b) Shift the iris aperture to the other end of the optical bench. Adjust the horizontal deviation of the laser beam by means of the thumb screw on the side of the plane mirror in such a way that the reflected beam passes through the opening of the aperture. The height of the beam should be nearly unchanged, if necessary re-adjust the vertical deviation of the plane mirror by means of the upper thumb screw until the beam passes through the aperture.

- Repeat actions a and b until the beam impinges on the aperture at the same place along the entire length of the optical bench. In the end return the iris aperture to upstream of the plane mirror.

- Push the plane mirror by approx. 0.8 cm towards the end of the bench. The laser beam will then shine at a distance of approx. 0.8 cm from the optical axis of the optical bench.

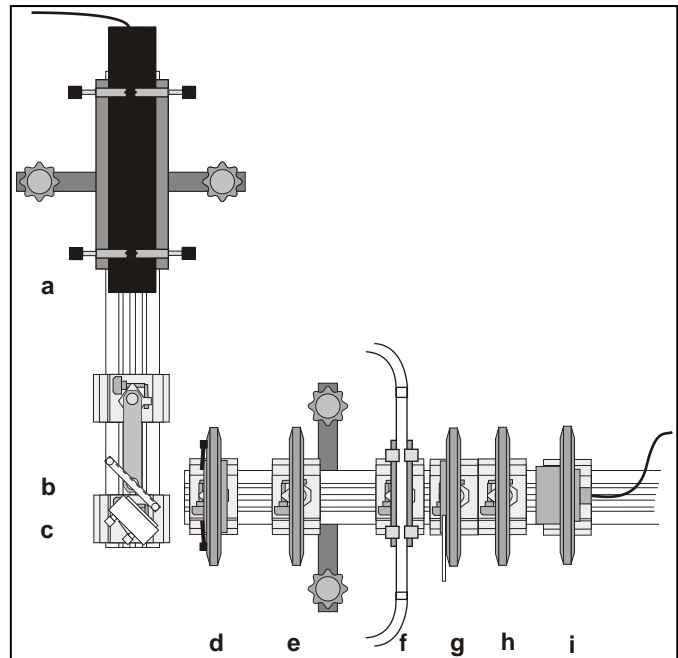


Fig. 5: Optical setup

a	He-Ne-laser head, 5 mW
b	beam divider 50% on cantilever arm
c	planar mirror with fine adjustment
d	ring diaphragm
e	lens in frame, $f = +100$ mm
f	glass tube
g	iris diaphragm
h	lens in frame, $f = +50$ mm
i	silicone photodetector

5. Aligning the beam divider so that the part of the beam reflected by the beam divider (short beam divider beam) is in parallel to the optical bench at a distance of approx. 1.6 cm to the part of the beam reflected by the plane mirror (short plane mirror beam):

- Insert the beam divider into the extension arm and set it up on the auxiliary bench upstream of the plane mirror by means of an optics rider so that the reflecting layer is pointing towards the laser, the beam divider is impinged on centrally and the reflected beam is directed in the direction of the laser. Adjust the beam divider by tilting it in such a way so that the ray is reflected back at the same height. To do this carefully loosen the Allan screws in the stalk and retighten appropriately (see operating instructions).

Attention: In addition to the main beam, further parasitical beams occur which originate from the reflections on the rear of the beam divider. During the adjustment ensure for that reason that the main beam (with the highest intensity) is used for the adjustment!

- Rotate the beam divider by 45° .
- Open the iris aperture sufficiently that the plane mirror beam on the right-hand side of the opening is no longer allowed to pass through (approx. 1.5 cm).
- Adjust the distance between beam divider and the plane mirror in such a way that the beam divider beam on the left-hand side of the iris aperture is still just blocked. The plane mirror beam must not be blocked by the beam divider holder.

- Adjustment of the beam divider beam in parallel to the plane mirror beam:

a) Shift the beam divider on the auxiliary bench in such a way that the reflected beam impinges on the left-hand side of the iris aperture symmetrically with the plane mirror beam.

b) Shift the iris aperture to the other end of the optical bench. Adjust the vertical deviation of the beam divider by carefully twisting the holder until the beam divider beam again impinges on the left-hand side of the iris aperture symmetrically with the plane mirror beam. Attention: The height of the beam should remain almost unchanged, if necessary readjust the tilt of the beam divider, so that the plane mirror beam and the beam divider beam impinge on the iris aperture at the same height.

c) Repeat actions a and b until the plane mirror beam and the beam divider beam shine in parallel to one another at a distance of approx. 1.5 cm over the entire length of the optical bench. Then remove the iris aperture.

6. Screening off the parasitical beams:

- Insert the ring aperture into the holder with the spring clamps and place it at the end of the optical bench as close as possible to the plane mirror and the beam divider, so that only the two main beams are allowed to pass through. All the parasitical reflections should be covered by the ring aperture.

7. Adjusting the superposition range of the plane mirror beam and the beam divider beam in the focus of the $f = + 100$ lens:

- Place the $f = + 100$ mm lens directly downstream of the ring aperture and adjust it vertically with respect to the optical axis. The centre of the lens should be located at the same height as the plane mirror beam and beam divider beam. The back reflections from the lens should be located at the same height and symmetrically with respect to the optical axis at the rear of the ring aperture.

- If required shift the translucent screen far enough that the beam again impinges on the screen.

- Insert the aperture with 3 diffraction holes into the holder with spring clamps and attach it downstream of the lens in such a way that the aperture is located precisely at a distance of $f = 100$ mm downstream of the lens. (Warning: The aperture is not located centrally above the holder.)

- Cover the plane mirror between the beam divider and the plane mirror with a sheet of paper. On the screen only the beam divider beam remains visible.

- Align the aperture in the holder in such a way that the beam passes centrally through the smallest opening in the aperture. This has been achieved when on the screen downstream of the aperture a symmetrical diffraction pattern from the lens appears.

- Then uncover the plane mirror beam again and by means of the thumb screws readjust the plane mirror in such a way that also this beam passes through the smallest aperture opening. For doing this no major modifications should be required.

- Then remove the aperture with 3 diffraction holes.

At the focus of the lens both beams will now superimpose coherently and form an interference pattern (see figure 4). This can now be projected by means of a further lens and be observed.

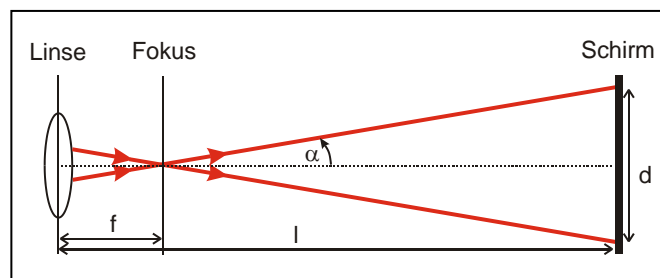


Fig. 6: Determination of the angle between the two beams

Projection of the interference zone for checking the adjustment:

- Place the $f = + 50$ mm lens approx. 15 cm downstream of the $f = + 100$ mm lens.
- Remove the translucent screen as far as possible (if possible even beyond the optical bench). Attention: maintain laser safety (see safety information)!
- Only shift the lens until the two beams are again superimposed on the screen.
- Rotate the screen so far that the beams impinge on it at a very small angle but are still superimposed.

On the screen now an interference pattern will become visible. For checking, cover each of the two beams in turn: The interference pattern will disappear. If both beams are uncovered again, the interference pattern should reappear.

Determination of the angle between the two beams

Before placing the tube into the path of the beam, the angle 2α has to be determined with which the two beams cross in the measuring volume.

- Remove the $f = + 50$ mm lens again from the optical bench.
- Place the translucent screen downstream of the $f = 100$ mm lens so that both beams impinge on it with as large a distance d between them as possible.
- Measure the distance d from beam centre to beam centre and the distance l between the lens and the screen.
- The required angle 2α is then

$$\tan \alpha = \frac{d}{2 \cdot (l - f)} \quad \text{where } f = 100 \text{ mm.}$$

Setting up the liquid reservoir

- Attach the stand rod onto the large stand base.
- Attach the separation funnel in the stand ring with clamp onto the stand rod
- Attach a silicone hose to the lower opening of the separation funnel.

Setting up the flow tube

- Carefully push the free end of the silicone hose as well as one end of the other silicone hose over the glass tube. Caution! The tube is made from quartz and is fragile.
- Attach the tube by means of the small clip plugs on the holder for plug-in elements and by means of an optics rider to the optical bench downstream of the $f = 100$ cm lens in such a way that the focus of the lens is located inside the tube.
- Then use a soft cloth and carefully wipe the dust and finger marks off the tube.

- Align the tube in the path of the beam in such a way that the tube is at right angles to the optical axis and the plane mirror beam and the beam divider beam impinge on it at mid-height. The return reflections from the glass surface are in this case symmetrical and at the same height as the incident beams.
- Push a Hofmann clip over each of the silicone hoses and use it to close the open end of the silicone hose.

Preparation of the liquid to be measured

The liquid to be measured is distilled water with a small quantity of scattering particles. The liquid to be measured may contain only very few scattering particles so that during the measurement there will be only one scattering particle in the measuring volume at any time.

- Mix a spatula tip of scattering particles in the beaker containing approx. 10 ml of distilled water.
- Fill the measuring beaker, clear SAN, with approx. 500 ml of distilled water.
- Add 2 to 3 ml of the mixture with the scattering particles and stir.
- Fill the liquid in the separation funnel and push the filled separation funnel to the upper end of the stand rod. Put the measuring beaker, clear SAN, under the open end of the silicone hose.
- Open the valve of the separation funnel fully. Open the Hofmann clip sufficiently that the hose with the tube completely fills with water. Then close the Hofmann clip.
- Again check the position of the tube. If necessary adjust the position so that the intersection point of the two beams is located at the centre of the tube now filled with water.

Setting up the photo-detector

- Set up the iris aperture approx. 5 cm downstream of the glass tube and open sufficiently that the two beams passing through the tube are just covered by the iris aperture. Now only light scattered in the tube will pass through the aperture.
- Immediately behind the iris aperture place the $f = 50$ mm lens, which will focus the scattered light onto the photo-detector.
- Place the silicon photo-detector downstream of the $f = 50$ mm lens in such a way that the light scattered in the tube is projected onto the silicon photo-detector.

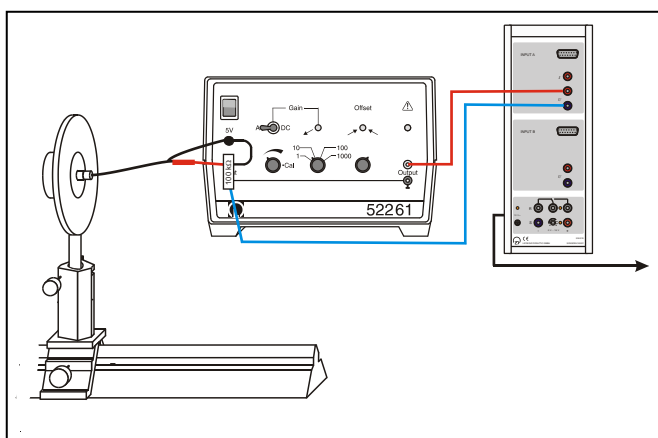




Fig. 7: Electrical setup

- Insert the blue 50 cm cable into the left-hand earth input of the AC/DC amplifier, the red 10 cm cable into the signal input, and connect both inputs to the 100 kΩ resistor (see figure 5).
- Connect the output of the photo-detector to the screened cable with the BNC/ 4 mm plug. Connect the large 4 mm plug of the screened cable to the 5 V output of the AC/DC amplifier and the small 4 mm plug of the screened cable via a coupling with the red cable to the signal input of the amplifier.
- Connect the output of the amplifier via the two 50 cm cables with the input A of Sensor-CASSY.
- Connect Sensor-CASSY to the computer.
- Select the AC/DC amplifier settings as follows:
 - AC/DC selection switch: AC setting
 - Continuous attenuation: Right-hand stop (calibrated)
 - Amplification selection switch: x 10

Carrying out the experiment

- Call CASSY Lab and load the settings.
- Carefully open the Hofmann clip on the lower aspirator bottle so that the liquid slowly flows through the tube. The flow can be observed on account of the light scattering by the scattering particles in the tube, the flow rate should be approx. several mm/s. This is achieved if the liquid drips slowly into the measuring beaker. Because of the low flow rate the water level in the separation funnel and therefore the flow rate will vary only very little. For this reason several minutes are available for one measurement.
- Start the measurement by clicking  or by pressing the F9 key. In the measuring parameters "repeating measurement" is set as the default. This automatically starts after each measurement the next one. In some measurements curves should be visible with a rapid modulation (see figure 8). This is the required signal. The measurement can then be stopped by clicking  or by pressing the F9 key.
- It is useful to set a trigger level in the measuring parameters above the noise level. Only larger modulations such as the required signal then activate the trigger and start the measurement.

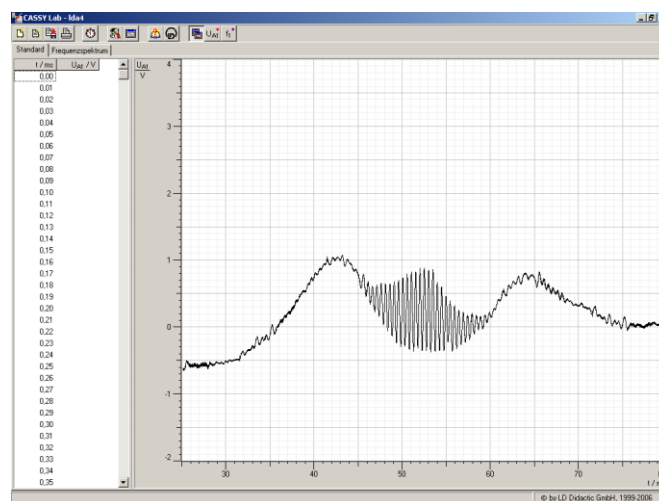


Fig. 8: Typical measuring data

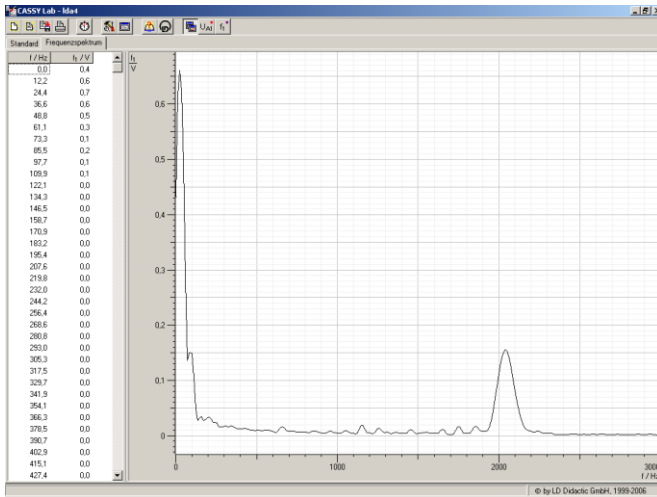



Fig. 9: Frequency spectrum for the measured curve from figure 8

- If the curve appears to be trimmed at its upper limit, the measuring range of the sensor has been chosen too small. In this case a larger measuring range must be selected in the sensor settings. If afterwards the curve still appears to be trimmed, the AC/DC amplifier may be overloaded. In this case a lesser amplification must be selected.
- In the frequency spectrum diagram the Fourier transformation of the measuring signal UA1 is available for the frequency analysis of the measuring signal. In addition to the signal with very small frequencies which is caused by, among other effects, scattering at the walls of the tube, electronic noise etc.; a sharp peak is apparent at higher frequencies (see figure 8). This is the required Doppler frequency ν_D .
- At very small flow rates (< 1 mm/s) it is possible that the frequency peak disappears into the range of low-frequency noise. In this case the Hofmann clip has to be opened little further.
- At very large flow rates (> 50 mm/s) turbulence can occur which can cause the flow rate to vary so strongly that the result is not very informative. In this case reduce the flow rate.
- In addition, the results can be falsified if the interval between two measuring points is chosen too large. In this case the maxima and minima of the curve are not represented properly (aliasing effect). In this case the interval has to be reduced in the measuring parameters.

Measurements at the tube centre for various flow rates

- When a good measurement is obtained (see figure 10) stop the measurement by clicking  or pressing the F9 key and save it. Determine the centre of the signal peak in the frequency spectrum. Then make a further measurement.
- Then repeat the measurement with different flow rates:
To obtain these, carefully open or close the Hofmann clip a little further. Only very small alterations are necessary! After the modification wait a further 2 min until the new flow rate has stabilised. Then record the measurement for the new flow rate, determine the centre of the peak in the frequency spectrum and store it.

Measurements in the centre of the tube and close to the wall of the tube at identical flow rates

- For observing the various flow rates in the centre of the tube and close to the wall of the tube carefully shift the tube on the optical bench so that the point of intersection of the two beams is now located close to the tube wall but still within the tube. In this position also record several curves, evaluate and store them.

Note: The measurements in the centre and close to the wall of the tube should be made at the same flow rate and therefore within a few minutes because otherwise the changing water level in the aspirator bottle will have an effect on the measurement. As a control, after the measurements close to the wall a further measurement in the centre of the tube is sensible so that the effect of the changing water level on the measurement can be estimated.

Measurement of the average flow rate by means of the increase in liquid contained in a measuring container and comparison with the measurement by means of LDA

- Carefully open the Hofmann clip on the silicone hose leading into the beaker so that the liquid drips slowly into the measuring beaker.
- Measure the time t required to fill the beaker with a certain quantity V of the liquid.
- Record parallel measuring curves in the centre of the tube (and close to the wall of the tube) and store them.
- Repeat for different flow rates.

Measuring example and evaluation

Determination of the angle between the two beams

For the determination of the angle 2α the following values were measured using the setup:

$$d = 24.1 \text{ cm}; l = 148 \text{ cm}.$$

Using these values, with $f = 100$ mm in equation (X)

$$\text{gives } \tan \alpha = \frac{d}{2 \cdot (l - f)} = 0.0873 \text{ and } \alpha = 4.99^\circ.$$

Measurements at the tube centre for various flow rates

Figure 10 shows a good measuring curve. For easier identification of the signal the trigger level was set to 0.1 V. Therefore the increase in the signal is not completely visible. Figure 10 shows a typical frequency spectrum measuring curve. The evaluation of the centre of the peak results in an average frequency of $\nu_{D1} = 319$ Hz.

Further measurements at different flow rates resulted in the following values:

$$\nu_{D2} = 1338 \text{ Hz and } \nu_{D3} = 3372 \text{ Hz}$$

If these values are used in equation (VII) for the speed component $v_{\perp} = \frac{\nu_D \cdot \lambda}{2 \sin \alpha}$, using the measured values above this results in $\lambda = 633$ nm:

$$\nu_{\perp 1} = 1.31 \frac{\text{mm}}{\text{s}}, \nu_{\perp 2} = 4.9 \frac{\text{mm}}{\text{s}} \text{ and } \nu_{\perp 3} = 12 \frac{\text{mm}}{\text{s}}.$$

As anticipated the values are of the order of several mm/s.

Measurements in the centre and close to the wall of the tube for identical average flow rates

The difference in the flow rates in the centre of the tube and at the wall of the tube on account of the parabola-shaped flow rate profile inside the tube can be demonstrated in the measurement.

Measurement at the centre of the tube: $v_M = 767 \text{ Hz}$

Measurement near the wall of the tube: $v_R = 376 \text{ Hz}$

This results in the flow-rates listed below:

$$v_{\perp M} = 2.79 \frac{\text{mm}}{\text{s}}$$

$$v_{\perp R} = 1.37 \frac{\text{mm}}{\text{s}}$$

The difference in the determined flow rates is clearly visible. In addition it is apparent, when several measurement are compared, that the scatter in the values measured close to the wall is much larger than at the centre of the tube. With this measuring method therefore a local measurement of the flow rate is possible.

Measurement of the average flow rate by means of the increase in liquid contained in a measuring container and comparison with the measurement by means of LDA

For comparing the average flow rates with the local flow rates the quantity of liquid flowing within a fixed time interval through the tube into a suitable container can also be measured. From the time t , the quantity of liquid V and the diameter of the tube D , the average flow rate \bar{v} can be determined.

Measuring time: $t = 120 \text{ s}$, quantity of liquid: $V = 25 \text{ ml}$, internal diameter of the tube: $D = 2r = 5.6 \text{ mm}$

With these one obtains

$$\bar{v} = \frac{V}{\pi r^2 \cdot t} = 8.5 \frac{\text{mm}}{\text{s}}$$

From the measuring curve (measurement at the centre of the tube) a Doppler frequency $\nu_D = 3302 \text{ Hz}$ and a flow rate $v_{\perp} =$

$12 \frac{\text{mm}}{\text{s}}$ are obtained at the same time.

The average flow rate is smaller than that determined by the LDA because the flow rate is measured locally with the LDA at the centre of the tube while by measuring the quantity of liquid the various flow rates in the tube are averaged.

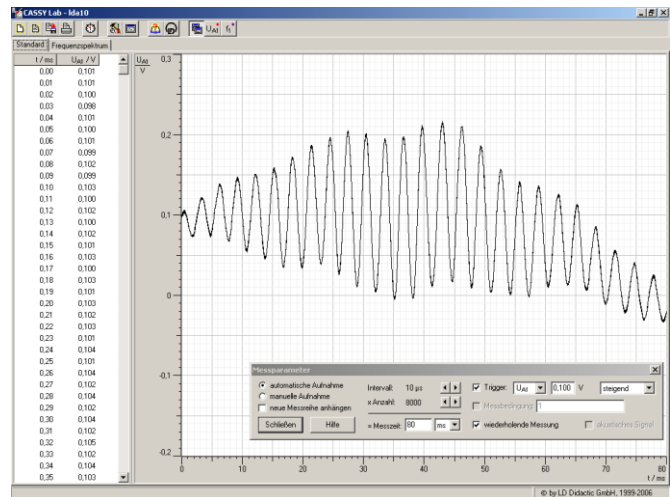


Fig. 10: Measuring example

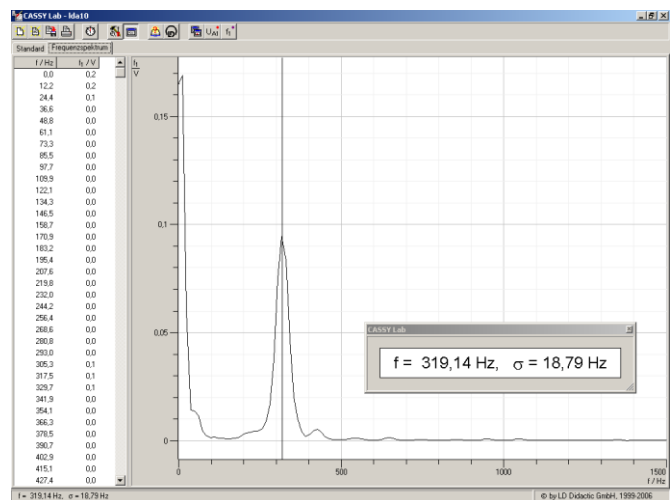


Fig. 11: Frequency spectrum for the measured curve from figure 10