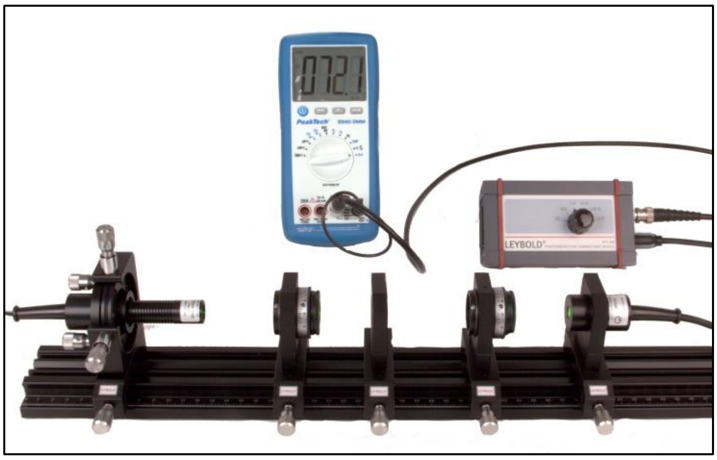
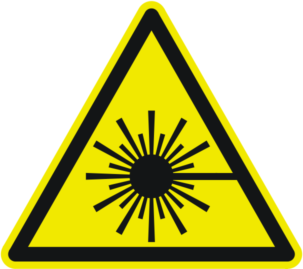


Polarization of light



Objects of the experiment

- Measuring the polarisation of different light sources
- Measuring Malus Law
- Investigation of optical activity
- Analyzing and quarter and half waveplates / Jones matrices

Safety notes

This experiment uses Lasers. According to EN 60825 they are rated class 3B.

474 5428:

Laser Class 3B, green 532nm, CW, <7 mW

Safety precautions are necessary. Please check with local regulations. Typically the use requires a safety sign and maybe a warning lamp that is on when the laser is activated and it might also be necessary to do and document a risk assessment. In some places it might be necessary to apply for a license or notify the relevant authorities.

Germany: According to OStrV it is necessary to inform the “Gewerbeaufsichtsamt” and the workers’ insurance “Berufsgenossenschaft” 14 days prior to startup.

Misuse of the lasers poses a health risk, especially for the eyes.

Do not operate the devices outside parameters specified in the manual.

People using the laser must be properly trained and students must be supervised.

As a general guidance, the user is advised to:

- Check the laser for damages before use
- Never to look into the beam
- Take necessary measures that no people or animals can accidentally enter the beam area
- Do not direct the beam on reflecting surfaces or into public areas
- Do not work close to the light path with reflecting tools
- Take off all jewelry and wristwatches when working with the laser to avoid reflections
- While placing or removing optical parts in the light path, switch off the laser or cover its exit
- Use laser protection glasses or laser adjustment glasses where necessary
- Supervise students by trained personnel when they work with the laser system
- Use the laser system only as described in the instruction manuals

Principles

a) Polarization of light

In this experiment the polarization of light will be investigated. Light is a three dimensional transversal electromagnetic wave usually with the electric and magnetic field vector perpendicular to the propagation direction. Polarization of an electromagnetic wave is defined by the direction of the electric field vector $\boldsymbol{E}$. It can be described by superposition of its two orthogonal components orientated in a plane perpendicular to the propagation direction.

$$E_x = E_{0x} e^{-i(\omega t - kz + \varphi_x + \varphi_0)}$$

$$E_y = E_{0y} e^{-i(\omega t - kz + \varphi_y + \varphi_0)}$$

These components are oscillating in time with the same frequency, however the amplitudes and phases may differ. This can be rewritten in a vector form with the fast oscillating term separated from the both components

$$\boldsymbol{E} = \boldsymbol{E}_0 \, e^{-i(\omega t - kz + \varphi_0)}$$

where

PJD 2014-12

$$\boldsymbol{E_0} = \begin{bmatrix} E_{0x}e^{i\varphi_x} \\ E_{0y}e^{i\varphi_y} \end{bmatrix}$$

The expression in the bracket is no longer dependent from the fast oscillating component, but contains only phase shifts and amplitudes. The vector is called **Jones vector** and reflects the polarization state. Since the relative amplitudes and phases fully determine the state of polarization, the Jones vector is a complete description of it.

A linear polarization is obtained if the relative phase $\boldsymbol{\varphi}_x - \boldsymbol{\varphi}_y$ is set to 0° or ± 180°.

An elliptical polarization will result for any other set of amplitudes and relative phases, including a circular case, when the amplitudes are equal and the relative phase is 90° ± 180°.

It is interesting to notice that for given amplitudes one can get a linear polarization in two directions depending on the phase difference set to 0° or 180°.

The difference of 180° in phase means that instead of a maximum of an electric field, a minimum is encountered, which results in a change of the direction of the electrical field vector.

Oftentimes, however, it is not necessary to know the exact amplitudes and phases of the vector components. Therefore Jones vectors can be normalized and common phase factors can be neglected. This results in a loss of information, but can greatly simplify expressions. For example, the following vectors contain varying degrees of information, but are all Jones vector representations for the same polarization state:

$$\begin{bmatrix} E_{0x}e^{i\varphi_x} \\ E_{0y}e^{i\varphi_y} \end{bmatrix} \to \begin{bmatrix} e^{i\varphi_x} \\ e^{i\varphi_y} \end{bmatrix}$$

In the following table shows the most common Jones vectors.

Polarization	Jones vector
linear, general	$\begin{bmatrix} \sin\alpha \\ \cos\alpha \end{bmatrix}$
linear, vertical (S)	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
linear, horizontal (P)	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
circular, ($+\boldsymbol{i}$ counterclockwise, $-\boldsymbol{i}$ clockwise)	$\begin{bmatrix} 1 \\ \pm i \end{bmatrix}$
elliptical, principal axes parallel to x, y axes	$\begin{bmatrix} A \\ \pm iB \end{bmatrix}$
elliptical, general	$\begin{bmatrix} A \\ B \pm iC \end{bmatrix}$

b) Optical elements and Jones matrices

A number of several optical elements influence the orientation of the polarization. The nature of their interaction with an optical wave allows to categorize those devices into three classes: **polarizers**, **phase retarders**, and **rotators**.

The transmission of a **polarizer** depends very strongly on the relative orientation of the polarizer and the incoming polarization of the light. In other words, a horizontally orientated polarizer will transmit all of the horizontally polarized light and none of the vertically polarized.

Phase retarders do not cut out any of the polarization components, instead of that they introduce a phase retardation between them. By changing the relative phase between the components a restrained control over the polarization can be achieved.

The last type, a **rotator**, turns the incident light polarization by some predefined angle. The rotators usually take the advantage of a property of optically active materials like quartz, or they rotate the whole beam spatially, so unlike retarders, the angle of rotation is not dependent from the incident polarization.

An effect of such elements on a light wave can be described by a matrix acting upon the Jones vector. The matrices that are multiplied to the Jones vector result in a new vector describing the polarization state after the element. These matrices are called **Jones matrices.**

A set of Jones matrices for a few elements is presented in the following table.

Component	Jones matrix
Horizontal (P) polarizer [PP]	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
Vertical (S) polarizer [PS]	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
Phase retarder	$\begin{bmatrix} e^{i\epsilon_x} & 0 \\ 0 & e^{i\epsilon_y} \end{bmatrix}$
Rotator	$\begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$

c) Quarter and Half Waveplates

The light traveling along a phase retarder will undergo the double refraction. More precisely, there is a slow and a fast optical axis. The electrical field oscillating along the fast axis will experience less retardation than in the case when it oscillates along the perpendicular, slow axis. The retardation of the fast and the slow axis are ϵ_x and ϵ_y, respectively, and when the difference between them is 180° (or π) the retarder is called a **half waveplate** and in case of 90° (or $\frac{\pi}{2}$) a **quarter waveplate**. A half wave plate in a beam of linearly polarized light rotates the polarization by the angle given by twice the difference between the fast axis and the orientation of the polarization.

To clarify the mechanism, one can imagine linear polarization of light as a superposition of two waves perpendicularly polarized along the fast and the slow axis of a waveplate. Introducing phase shift of a half wave (180° or) will result in a change of polarization. The angle of rotation depends on the amplitudes of these components along the waveplate axes. Applying the Jones formalism, it shows the effect of a half waveplate with the fast axis orientated parallel to the table (0°) on a parallel (P) polarized light.

$$\begin{bmatrix} e^{-i\pi/2} & 0 \\ 0 & e^{i\pi/2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix}$$

The outcome is a vector representing exactly the same P polarization multiplied by a factor $-i$ which is due to the phase shift of 180° experienced by passing the plate. This example shows that if the fast axis is parallel to the polarization no rotation occurs. Corresponding Jones matrices for rotated half waveplate $HW[\beta]$ and quarter waveplate $QW[\beta]$ can be found by applying the rotation operator on a waveplate.

$$HW[\beta] = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} e^{-i\pi/2} & 0 \\ 0 & e^{i\pi/2} \end{bmatrix} \begin{bmatrix} \cos -\beta & -\sin -\beta \\ \sin -\beta & \cos -\beta \end{bmatrix}$$

$$QW[\beta] = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} e^{-i\pi/4} & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \begin{bmatrix} \cos -\beta & -\sin -\beta \\ \sin -\beta & \cos -\beta \end{bmatrix}$$

where β is the angle of rotation.

For example

$$HW[45°]P = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -i \end{bmatrix}$$

Multiplying this matrix by *P* polarization has the effect of rotating it 90° off the plane of the electrical field oscillation, as expected.

d) Malus Law

When unpolarized light is incident on an ideal polarizer, only the electric field vector's component in the direction of polarization can pass through. This leads to a reduction in the intensity of the transmitted light by one-half of the incident light. If this linear polarized light is transmitted through a second polarizer (so called analyzer) at an angle α to the first, only part of the electric field vector $E = E_0 \cos\alpha$ can get through.

Because the intensity of light I is proportional to the square of the electric field vector E.

$$I(\theta) = I_0 \cos^2\theta$$

This equation is called Malus Law.

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Experiments

To visualize and measure the properties of polarization and optical elements these experiments are presented:

Experiment 1:

In the first experiment the polarization sate of the Diode Laser Module, 532 nm is recorded.

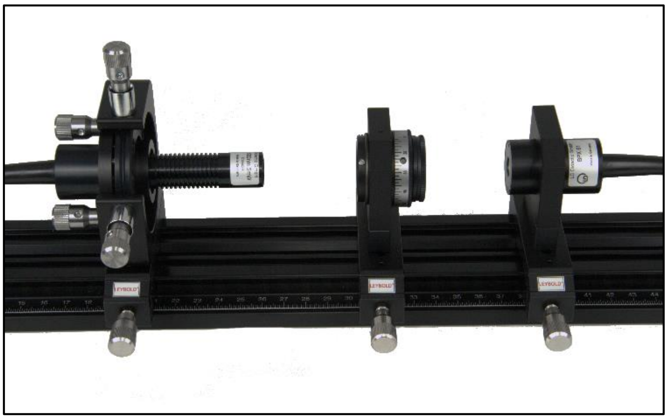

Fig. 1: Setup to measure the polarization of the laser diode.

Setup on the Bench:

1	Profile rail, 500 mm	........474 5442
1	Adjustment holder, 4 axes, with stop ang	......474 2112
1	Diode Laser Module, 532 nm	........474 5418
1	Polariser / Analyser with Rotator	........474 1124
1	Mounting Plate C25 with Carrier 20 mm	.......474 209
1	Si PIN Photodetector	........474 321
1	Photodetector signal conditioning box	........474 306
1	Digital multimeter DMM 121	........531 173
1	Screened cable, BNC/4 mm	........575 24

On the bench, the laser diode is put into the adjustment holder. The polarizer with rotator is put on behind the laser diode. Set the rotator in the vertical position (angle = 0°). At the end of the bench the photodetector is placed and connected with the photodetector signal conditioning box and with the digital multimeter via BNC cable.

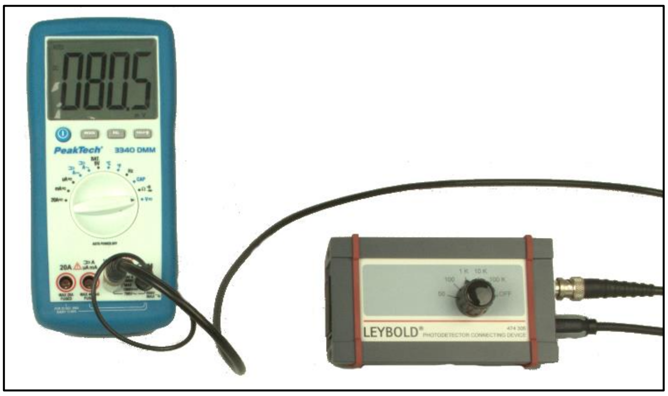

Fig. 2: Measuring setup and connections.

Run the multimeter in voltage mode and set the controller of the conditioning box so that the output voltage is around about a few hundred mV. Note that the conditioning box runs with a 9 V battery so that this is the maximum output voltage.

Now turn the rotator of the polarizer to measure the intensity depending on the angle.

Results:

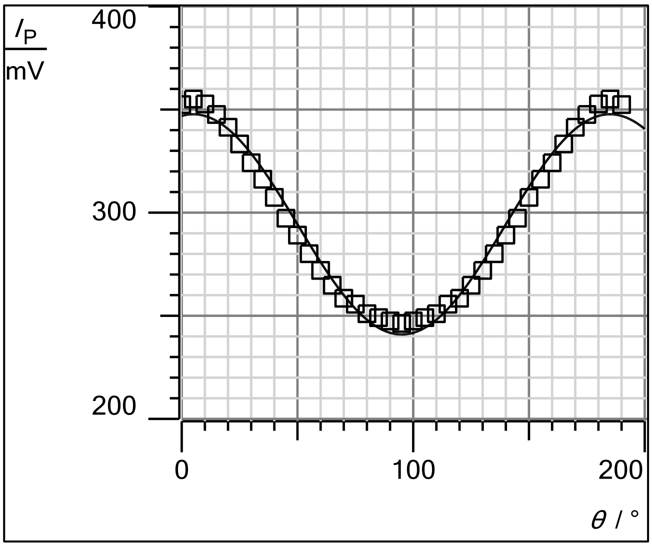

Fig. 3: Polarization of the laser diode

In Fig. 3 is shown that the intensity minimum is not equal to zero at $\theta \approx 90°$. Obviously, the laser beam is not fully linear polarized, but contains either an orthogonal linear component or circular polarized light.

The green laser is a so called diode pumped solid state laser.

Experiment 2:

In this experiment the laser is replaced by one of the LED either white or red.

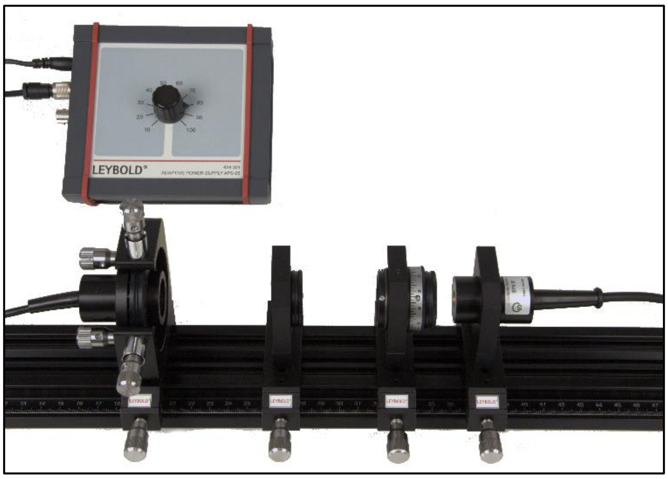

Fig. 4: Measuring the polarization of white and red LED.

Setup on the Bench:

1	Profile rail, 500 mm	........474 5442
1	Adaptive Power Supply	........474 301
1	Adjustment holder, 4 axes, with stop ang	.....474 2112
1	LED Lamp, White	........474 5411
1	LED Lamp, Red	........474 5412
1	Plano-Convex lens f = 40 mm, C25 mount	....474 5216
1	Polariser / Analyser with Rotator	........474 1124
2	Mounting Plate C25 with Carrier 20 mm	.......474 209
1	Si PIN Photodetector	........474 321
1	Photodetector signal conditioning box	........474 306
1	Digital multimeter DMM 121	........531 173

1	Screened cable, BNC/4 mm	.... 575 24

On the bench, the laser diode is replaced by one of the LED lamp. The LED lamp is connected to the adaptive power supply which control the intensity of the LED lamp. To collimate the emitted light of the LED a collimating lens is placed behind the LED so that the beam is focused to infinity.

Results:

The measured results are plotted against the angle of the polarizer which can be set in steps of 5 degrees.

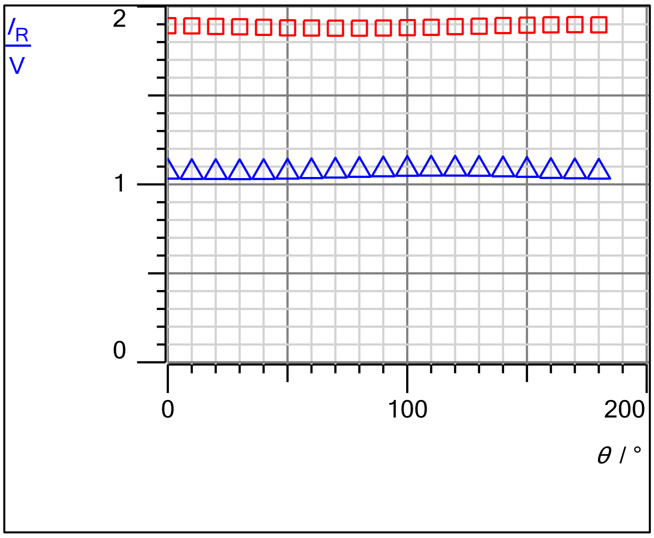

Fig. 5: Polarization of the LED. White: red squares. Red: blue triangle.

In Fig. 5 is shown that the LED lamps are completely unpolarized, as expected.

Experiment 3:

In this experiment the Malus Law is investigated.

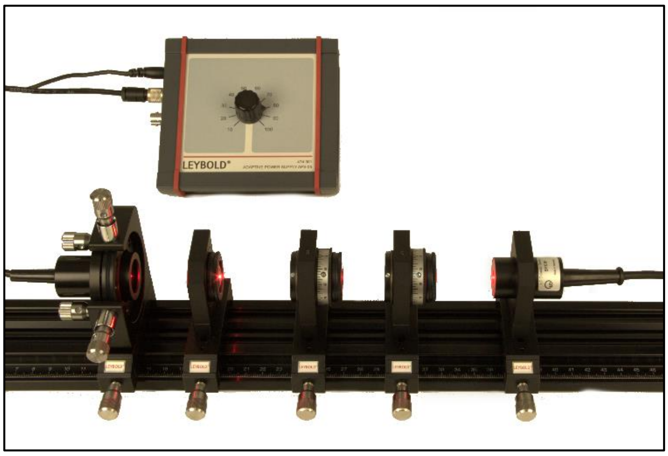

Fig. 6: Setup for measuring Malus law

Setup on the Bench:

1	Profile rail, 500 mm	.... 474 5442
1	Adaptive Power Supply	.... 474 301
1	Adjustment holder, 4 axes, with stop ang	..... 474 2112
1	LED Lamp, White	.... 474 5411
1	LED Lamp, Red	.... 474 5412
1	Plano-Convex lens f = 40 mm, C25 mount	.... 474 5216

2	Polariser / Analyser with Rotator	474 1124
2	Mounting Plate C25 with Carrier 20 mm	474 209
1	Si PIN Photodetector	474 321
1	Photodetector signal conditioning box	474 306
1	Digital multimeter DMM 121	531 173
1	Screened cable, BNC/4 mm	575 24

Place another polarizer behind the first polarizer. The second one is also called analyzer. Set the first polarizer in the vertical direction ($\theta = 0^\circ$) and the analyzer in the same vertical direction ($\theta = 0^\circ$), shown in Fig. 7.

The polarizer can be readjusted by loosening the small screw, if there is not the minimum of intensity at the setup shown in Fig. 8.

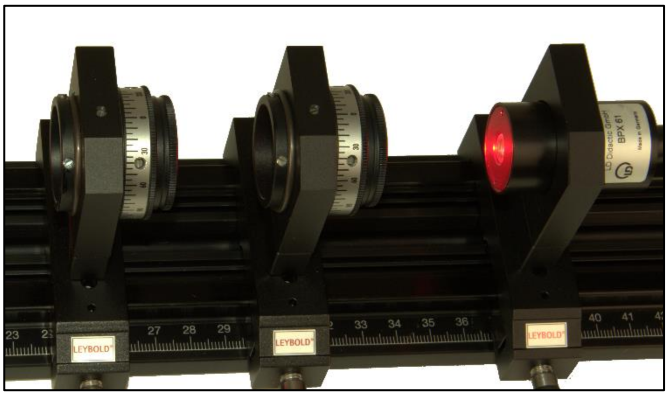

Fig. 7: Polarizer and analyser in vertical direction. Light can travel through both.

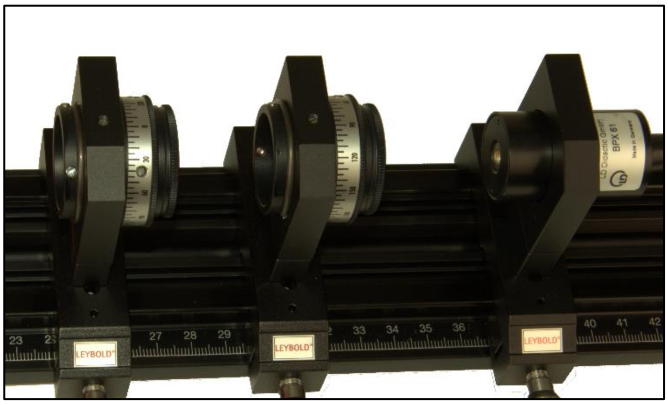

Fig. 8: Polarizer ($\theta = 0^\circ$) and analyser ($\theta = 90^\circ$) in crossed directions. No light can travel through both.

Results:

The measured intensities are plotted against the angle of the polarizer shown in Fig. 9. The function of Malus Law

$$I(\theta) = I_0 \cos^2 \theta$$

fits perfectly to the measured data.

The measurements can be repeated with the other LED and the laser diode and the results are the same.

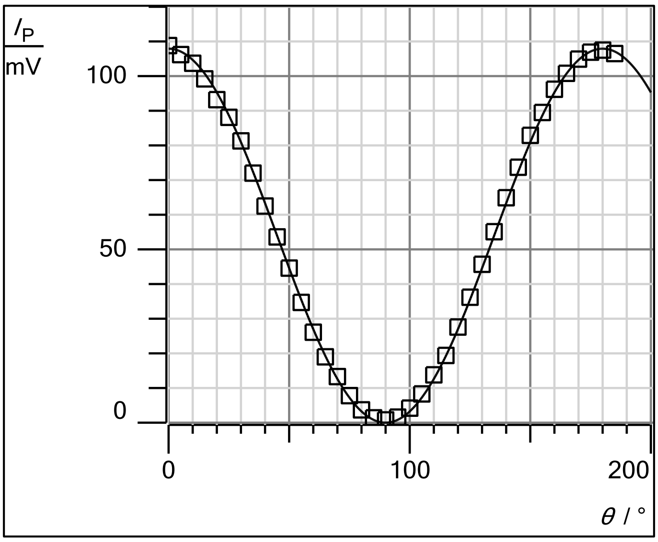

Fig. 9: Verifying Malus Law.

Experiment 4:

In this experiment optical elements as quartz, quarter and half wave plates are investigated.

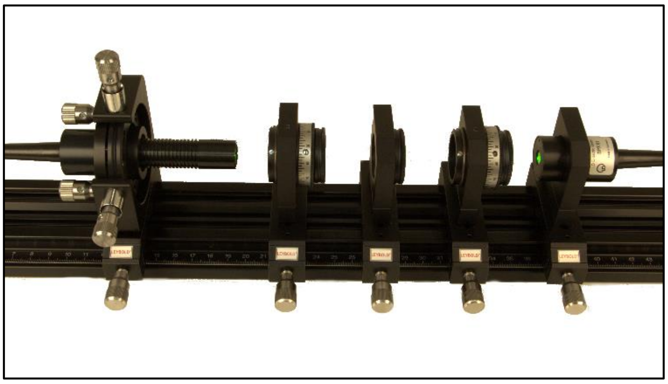

Fig. 10: Setup for optical active elements: quartz, quarter or half wave plates.

Setup on the Bench:

1	Profile rail, 500 mm	474 5442
1	Adaptive Power Supply	474 301
1	Adjustment holder, 4 axes, with stop ang	474 2112
1	Diode Laser Module, 532 nm	474 5418
2	Polariser / Analyser with Rotator	474 1124
2	Mounting Plate C25 with Carrier 20 mm	474 209
1	Optical Quartz Plate in C25 Mount	474 5260
1	Quarter wave plate, C25	474 5320
1	Half Wave Plate, C25 Mount	474 5275
1	Si PIN Photodetector	474 321
1	Photodetector signal conditioning box	474 306
1	Digital multimeter DMM 121	531 173
1	Screened cable, BNC/4 mm	575 24

Replace the LED lamp by the laser diode. Set between polarizer and analyser a mounting plate C25 where a quartz plate, a quarter or a half wave plate can be inserted.

a) Quartz plate

Turn the polarizer and analyser in the crossed direction, so that no light of the laser diode can travel through and the intensity is at a minimum. Insert the quartz plate and observe the chance in intensity.

This effect can be investigated more precisely. Therefor set the polariser and analyser in the vertical direction and place the quartz plate in the middle. The analyser can be turned in steps of 5 degrees and the intensity is measured.

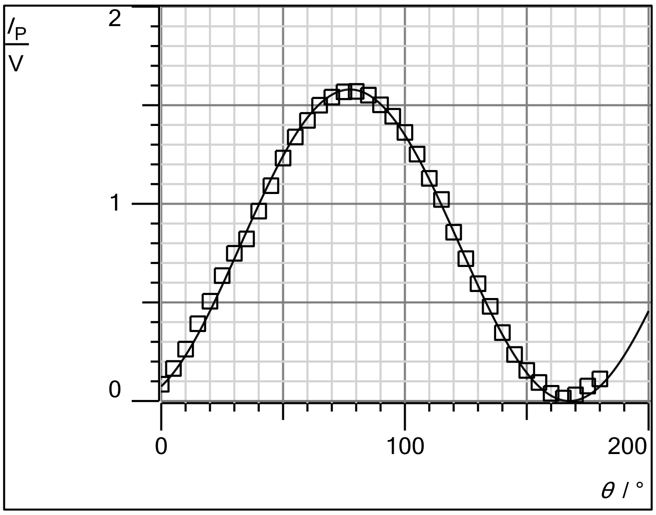

Fig. 11: Measuring the optical activity of a quartz plate.

In Fig. 11 the effects of the optical activity of a quartz plate is shown. The direction of polarization has been changed. By fitting a curve at the measured data the turning is angle can be determined. This quartz plate has a turning angle of about 77°.

To see if the effect depends on the way the light travels through the quartz plate, reverse the quartz plate so that the light can travel through the quartz plate the other way around.

The experiment shows that there is the same effect and it is independent on which way the light travels through the quartz plate, forward or backward. This is called reciprocity.

Rotate the Quartz plate around the optical axis. The result of changed polarization will remain the same. There is no dedicated optical axis visible.

To be precise, the optical axis of this quartz plate is along the optical axis of the setup. The quartz plate is a different phase retarder for left and right handed circular polarized light.

Depending on the atomic structure, there are two different types of Quartz crystals, one will delay left circular light while the other will delay the right circular light.

Using linear polarized light in the example, one kind of quartz will rotate the polarisation clockwise, while the other one will rotate it counterclockwise.

b) Quarter wave plate

Set the polarizer and analyser in the crossed position. Substitute the quartz plate with the quarter wave plate. Turn the quarter wave plate so that the intensity is at a minimum.

The axis of polarization of the laser light is so that it makes a 0° with the fast or slow axes of the quarter wave plate. The polarization is not changed and remains linear.

The Jones matrices and vectors leads to

$$PPQW[0°]P = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}e^{-i\pi/4} & 0\\ 0 & e^{i\pi/4}\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

So there is no intensity, as measured.

Turn the first polarizer about 45°. The axis of polarization of the laser light is now so that it makes a 45° with the fast or slow axes of the quarter wave plate

$$QW[45°] = \frac{\sqrt{2}}{2}\begin{bmatrix}1 & -i\\ -i & 1\end{bmatrix}$$

By multiplying the Jones vector P

$$QW[45°]P = \frac{\sqrt{2}}{2}\begin{bmatrix}1 & -i\\ -i & 1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = \frac{\sqrt{2}}{2}\begin{bmatrix}-i\\ 1\end{bmatrix}$$

the outcome wave is circular polarized. To proof this the analyzer is turned at any position and the measured intensity should not change.

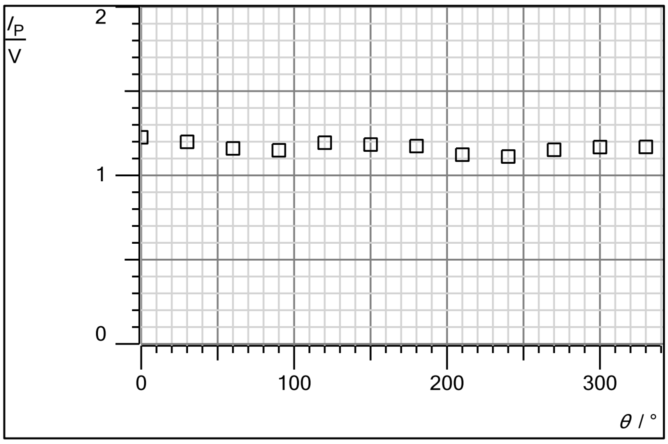

Fig. 12: Measuring the polarization of the wave after the quarter wave plate.

The measured intensities do not change by turning the analyzer as shown in Fig. 12. In this experiment the direction of light entering the quarter wave plate, forward or backward, does not affect the results.

Again, this a Quartz plate, but this time the crystal was rotated by 90° before cutting so we do have the optical axis perpendicular to the axis of the experiment. Rotating the quarter wave plate out of the 45° position relative to the first polarizer will change the result.

c) Half wave plate

Set the polarizer and analyser in the vertical position. Substitute the quarter wave plate with the half wave plate. Turn the wave plate so that the intensity is at a minimum.

The axis of polarization of the laser light is so that it makes a 45° with the fast or slow axes of the half wave plate. The polarization is turned 90° but remains linear.

The Jones matrices and vectors leads to

$$PSHW[45°]P = \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}0 & -i\\ -i & 0\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

So there is no intensity, as measured.

By turning the analyzer the intensity should raise and follow a $\sin^2$-function.

Again, this half-wave plate is quartz plate cut the same way as a quarter waveplate. It could be simply made twice as thick, but both are so called high order waveplates.

The true delay is not 45° but 45°+n*360°.

A plate with n=0 would be very thin and fragile.

The same is true for the half waveplate, so they both will not differ much in thickness.

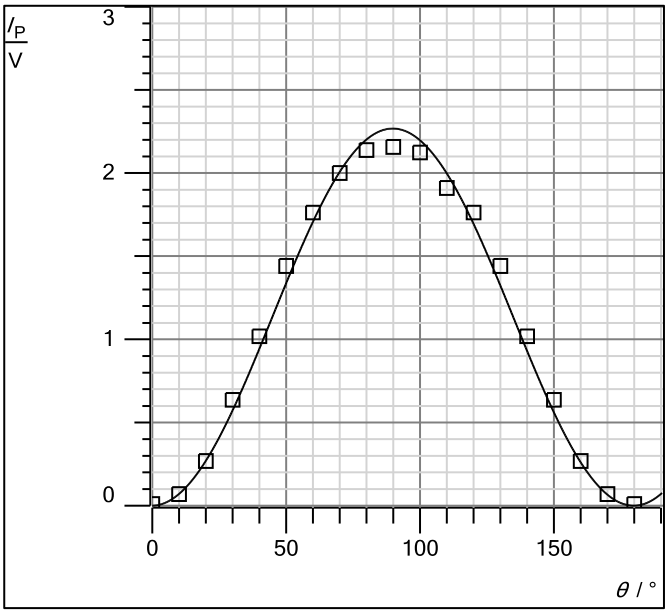

Fig. 13: Measuring the intensity after a half wave plate.

Fig 13. shows a fit to the measured intensities. The maximum is at $= 90°$. This proofs that the half wave plates turns the direction of polarization by 90°.

Apparatus

1	Plano-Convex lens f = 40 mm, C25 mount	474 5216
1	Optical Quartz Plate in C25 Mount	474 5260
1	Quarter wave plate, C25	474 5320
1	Half Wave Plate, C25 Mount	474 5275
2	Polariser / Analyser with Rotator	474 1124
1	Photodetector signal conditioning box	474 306
1	Si PIN Photodetector	474 321
1	Digital multimeter DMM 121	531 173
1	Screened cable, BNC/4 mm	575 24
1	Adaptive Power Supply	474 301
1	LED Lamp, White	474 5411
1	LED Lamp, Red	474 5412
1	Diode Laser Module, 532 nm	474 5418
1	Profile rail, 500 mm	474 5442
2	Mounting Plate C25 with Carrier 20 mm	474 209
1	Adjustment holder, 4 axes, with stop ang	474 2112
1	Transport and Storage Box #01	474 251
1	Manual Polarisation of light	474 7206