Determining the velocity of light by means of the rotating-mirror method according to Foucault and Michelson

Measuring the image shift as a function of the rotational speed of the mirror

Objects of the experiment

- To measure the shift Δx of the image of the light source for a rotary mirror
- To measure the velocity ν of the rotary mirror
- To determine the velocity of light c from the rotational speed ν, the shift Δx and the light path Δs between the rotary mirror and the end mirror.

Principles

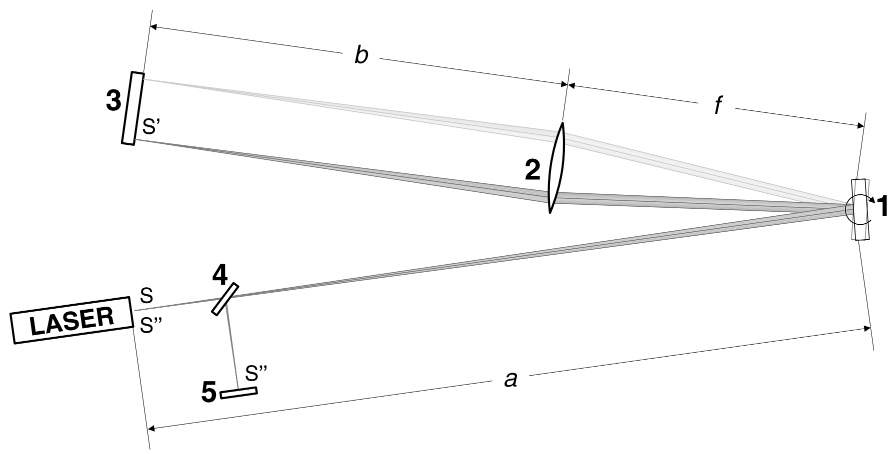
Measurement of the velocity of light by means of the rotary mirror method utilizes a concept first proposed by L. Foucault in 1850 and perfected by A. A. Michelson in 1878. In this method, light travels along a known path in both directions between two mirrors; the transit time required for this is initially unknown. During this time, the first mirror rotates at a known, very high frequency. Consequently, the returning light beam is incident on the rotary mirror at an altered angle, and is also reflected by this mirror. The position of the reflection is read from the scale, and this value is used to calculate the rotational angle of the mirror, and thus the transit time of the light.

In the experiment setup described here, the lamp used as a light source in earlier versions has been replaced by an He-Ne laser (see Fig. 1). The light source S is placed at a distance *a* from the rotary mirror **(1)**. The light reflected by this mirror is incident on lens **(2)**, which is set up at a distance equivalent to its focal length *f*. This produces an image S' of the light source on one of the planar end mirrors **(3)** set up at a distance *b* from the lens. The reflection of the light beams at the end mirror and their re-reflection at the rotary mirror produces an image S'' of the original image S' at the emission aperture of the laser.

When the rotary mirror turns, image S' moves across the end mirror. Between the lens and the end mirror, the main beam runs parallel to the axis of the lens, because the rotary mirror is in the focal point of the lens. At the end mirror, the main beam is reflected into itself, and returns along the same path to the rotary mirror, and from there to the emission aperture of the laser. The image S'' is always in the same location, regardless of the position of the rotary mirror, when the mirror is stationary or turning slowly. The images S' and S'' are generated for as long as the primary beam from the rotary mirror is within the aperture angle of the lens. Thus, when the mirror is rotating, the brightness of images S' and S'' is greater, the larger the usable width of the lens is.

A beam splitter **(4)** reflects a part of the returning light onto a glass scale **(5)**. The optical path from the rotary mirror to the glass scale is also *a*, so that image S' also appears on the glass scale.

Fig. 1: Beam path for determining the velocity of light using the rotary-mirror method
1 Rotary mirror
2 Lens, f = 5 m
3 End mirror
4 Beam splitter
5 Glass scale



Apparatus

1 Rotary mirror with motor, for determining the velocity of light	476 40
1 He-Ne laser, linearly polarized	471 840
1 Front silvered mirror, 120 mm dia.	463 20
1 Lens, f = 5 m	460 12
1 Beam splitter	471 88
1 Holder with spring clips	460 22
1 Glass scale	311 09
1 Variable low-voltage transformer, 0 to 250 V	521 40
1 Two-channel oscilloscope 303	575 211
1 Semiconductor detector	559 92
1 BNC cable, 1 m	501 02
1 Straight, BNC	501 10
1 Stand rod, 100 cm	300 44
1 Stand rod, 47 cm	300 42
1 Stand rod, 25 cm	300 41
1 Stand base (large), V-shape	300 01
4 Stand bases (small), V-shape	300 02
1 Saddle base	300 11
2 Leybold multiclamps	301 01
1 Bosshead	301 09
1 Wooden ruler, 1 m	311 03

In the time Δt which the light requires to travel the path

$$\Delta s = 2\,(f + b) \qquad \text{(I)}$$

from the rotary mirror to the end mirror and back, the rotary mirror rotating at a high value for ν turns by the measurable angle

$$\Delta\alpha = 2\,\pi\,\nu \cdot \Delta t \qquad \text{(II).}$$

This causes a shift in the position of image S” on the glass scale by the amount

$$\Delta x = 2\,\Delta\alpha \cdot a \qquad \text{(III).}$$

Thus, we may say:

$$c = \frac{\Delta s}{\Delta t} = \frac{8\pi\nu \cdot (f + b) \cdot a}{\Delta x} \qquad \text{(IV).}$$

The distances a and b cannot be set to just any values independently of each other. As the light source is sharply imaged on the end mirror, the law of imagery applies:

$$\frac{1}{f} = \frac{1}{b} + \frac{1}{a + f} \qquad \text{(V).}$$

Safety note

The He-Ne laser fulfills the German technical standard “Safety Requirements for Teaching and Training Equipment – Laser, DIN 58126, Part 6” for class 2 lasers. When the precautions described in the Instruction Sheet are observed, experimenting with the He-Ne laser is not dangerous.

- Never look directly into the direct or reflected laser beam.
- Do not exceed the glare limit (i.e. no observer should feel dazzled).

Consequently, for the velocity of light, we ultimately obtain the relationship

$$c = 8\pi \cdot (f + 2a)\, f \cdot \frac{\nu}{\Delta x} \qquad \text{(VI).}$$

Thus, producing the greatest possible shift Δx requires the highest possible speed ν and the longest possible focal length f and distance a. The rotary mirror for determining the velocity of light (476 40) has a speed of ν = 450 Hz, and the lens (460 12) has a focal length f = 5 m. In principle, any value can be selected for the distance a. However, one must then be willing to accept intensity losses due to the divergence of the laser beam, as well as a more painstaking adjustment process. The arrangement shown in Fig. 1 represents a viable compromise, in which the laser and the end mirror are positioned side by side, i.e. the relation $a = f + b$ applies. By inserting equation (V), we can calculate

$b = \sqrt{2} \cdot f = 7.1$ m and

$a = (1 + \sqrt{2}) \cdot f = 12.1$ m

The distance a between the laser and the rotary mirror corresponds to the overall length of the setup.

Setup

Adjustment of the optical components is much easier when the arrangement is set up by two persons. The beam path should be as horizontal as possible.

Select an area with a flat floor surface, and clamp the rods of the optical components in the apex of the stand bases (see Fig. 2) so that they extend about 1 cm from the bottom of the stand bases and touch the floor (beam height: 15 cm).
If the floor is noticeably uneven, clamp the rods in the middle strut of the stand bases, and leave some play for height correction.
Check the beam height with a ruler.

Laser, lens and end mirror:

- On the floor, mark position $P_1 = 0$ for the rotary mirror, P_2 = 5 m for the lens and P_3 = 12.1 m for the end mirror and laser (see Fig. 1).
- Mount the laser on the rod and tape a piece of white paper around the emission aperture; this will make subsequent observation of the returning laser beam much easier.
- Clamp the rods of the laser, the lens and the end mirror into one small stand base each.
- Align the rods as precisely vertical as possible using the adjusting screws on the stand bases.
- Set up the laser and the end mirror next to each other at P_3, with a distance of 30–40 cm between midpoints (see Fig. 1 and Fig. 2).

Rotary mirror:

- Clamp the stand rod at the apex of the stand base so that it extends about 1 cm from the bottom of the stand base and touches the floor.
- Attach the rotary mirror for determining light velocity to the stand rod so that the center of the mirror is on the level of the beam (see Fig. 3). Check the beam height with a ruler.
- Set up the rotary mirror at point P_1 and align the rotational axis of the mirror as precisely vertical as possible using the adjusting screws on the stand base. Do not insert the adjusting wrench yet.

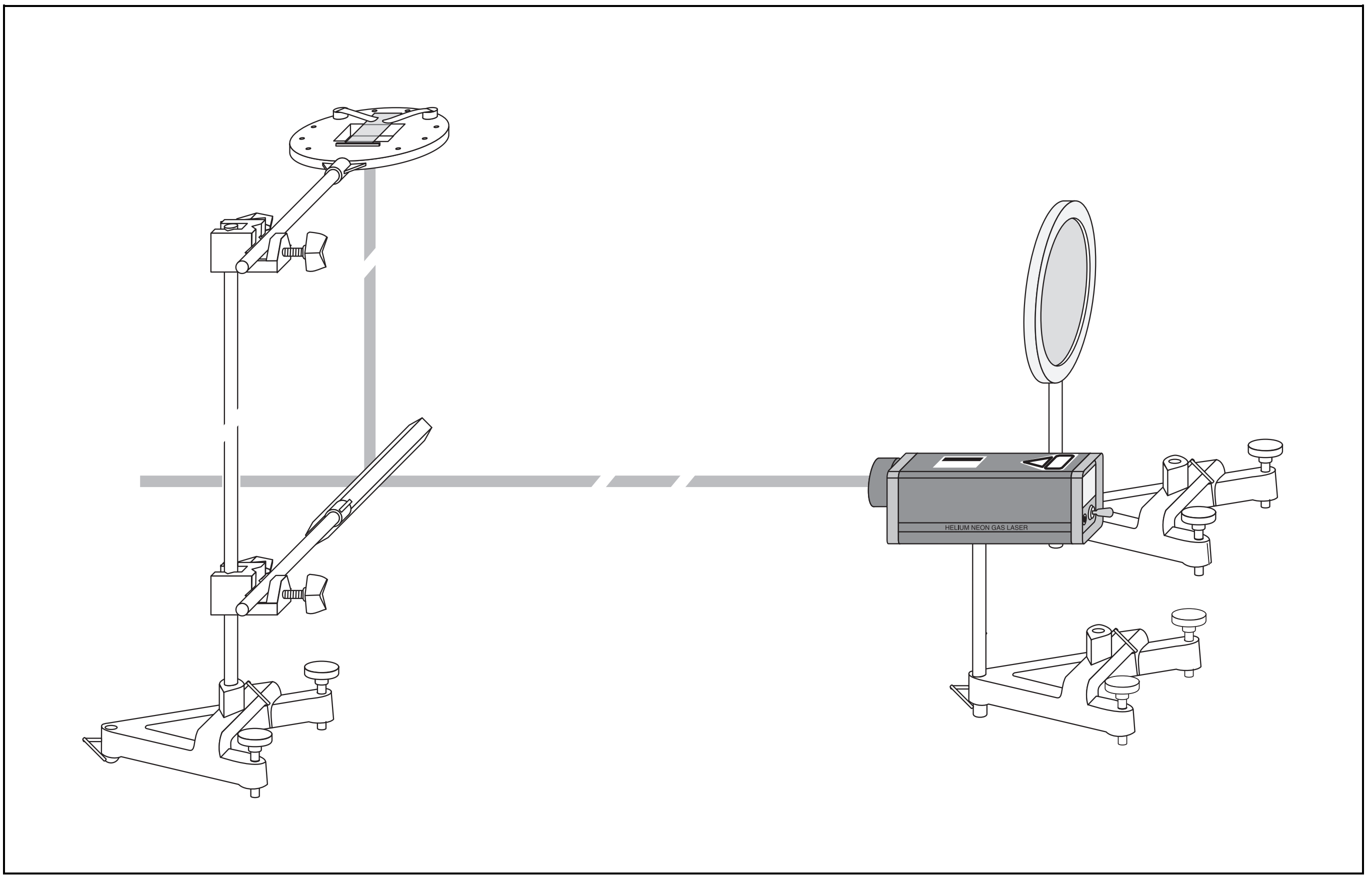
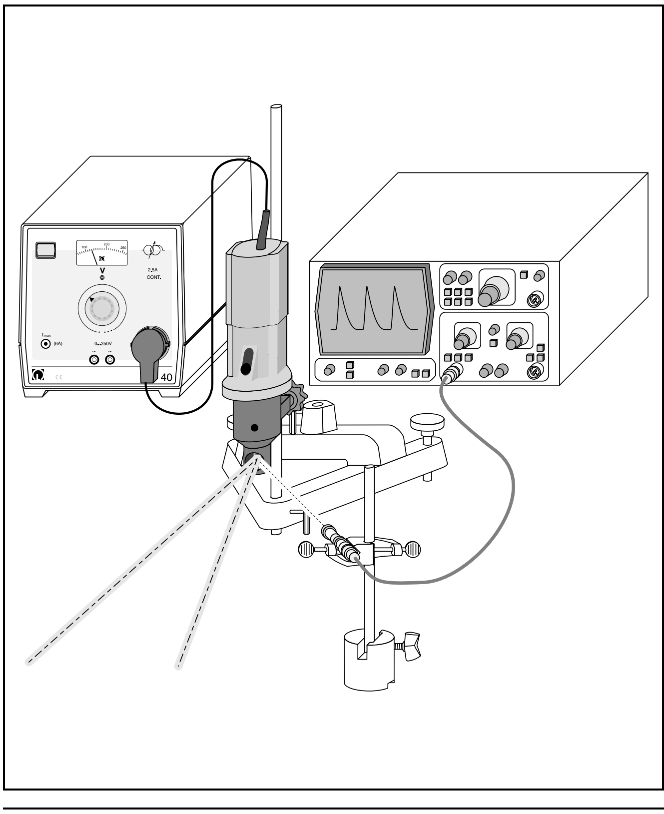


Fig. 2: Setup section 1 with laser, end mirror, beam splitter and glass scale; distances between the apparatus are not drawn to scale.

Fig. 3: Setup section 2 with rotary mirror, variable low-voltage transformer and setup for frequency measurement



– Connect the motor of the rotary mirror to the variable low-voltage transformer and turn the adjusting knob of the transformer to 0 V.
– Switch on the motor and the variable low-voltage transformer.
– Weigh down the stand base so that it cannot slide.
– Slowly increase the output voltage of the variable low-voltage transformer to 240 V and check whether the stand base of the rotary mirror moves.

Adjustment:

– Reduce the output voltage of the variable low-voltage transformer to 0 and switch this unit off.
– Using the adjusting wrench, turn the mirror side to the front.
– Switch on the laser and, by turning and moving the stand base and varying the adjusting screws, align the laser beam so that the rotary mirror is symmetrically illuminated.
– Using the adjusting wrench, turn the rotary mirror until the laser beam is reflected on the end mirror (initially without the lens). Use the adjusting screws on the stand base to position the reflected beam on the center of the end mirror. If necessary, carefully turn and move the stand base.
– Readjust the laser as necessary.
– Set up the lens at position P_2 so that it is perpendicular to the beam in the beam path between the rotary mirror and the end mirror (see Fig. 1). Align the lens so that the light beam passes through the center of the lens and strikes the center of the end mirror.
– Hold a sheet of paper between the lens and the end mirror and check whether the smallest beam diameter is at the mirror, i.e. image S’ is sharply focused; you may need to remove the end mirror.

If the image is not sharp, carry out the following steps (the cross-shaped interference pattern, in the origin of which the image rests, is due to diffraction at the edges of the rotary mirror, and is unavoidable):

– Remeasure positions P_2 and P_3 and correct the setup.
– By carefully turning the stand base and varying the adjusting screws, align the end mirror so that it reflects the light beam through the center of the lens precisely on the rotary mirror.
– Turn one of the adjusting screws on the end-mirror stand base one turn inward and outward and check whether the returning light beam falls on the laser emission aperture, and whether the diffraction pattern formed together with image S” is symmetrical with respect to the aperture.
– If necessary, correct the alignment of the end mirror.

Fine adjustment:

– Using the adjusting wrench, slowly turn the rotary mirror and check whether the reflected beam passes horizontally through the middle of the lens and the end mirror, and whether image S” remains stable during this process.
– If necessary, correct the alignment of the rotary mirror using the adjusting screws on the stand base.
– Then, turn the rotating mirror back to the initial position.
– Readjust the lens and end mirror as necessary.

Beam splitter and glass scale:

– Mount the beam splitter **(4)** and holder with spring clips **(5)** and place them in the setup in the beam path between the rotating mirror and the laser (see Fig. 1 and Fig. 2; distance from beam splitter to laser aperture = distance from beam splitter to holder).
– Turn the beam splitter to face 45° upwards, so that the beam coming from the rotary mirror is reflected straight up.
– Attach the glass scale in the holder with spring clips so that the reflected beam is incident approximately in the middle. Use a piece of paper as an adjusting aid, and correct the alignment of the beam splitter and the holder as necessary.

If the laser beam no longer falls on the center of the rotary mirror after the beam splitter is set up:

– Readjust the alignment of the laser.

Test run and frequency measurement:

– Attach the semiconductor detector at the level of the beam and connect it to the oscilloscope using the BNC cable (see Fig. 3).
– Place this element as close as possible beside the rotary mirror, so that the light beam passes across the semiconductor detector when the mirror turns, and no shadow is cast on the beam path.
– Remove the adjusting wrench from the rotary mirror. Switch on the variable low-voltage transformer and slowly increase the output voltage to 40–50 V.
– Display the output signal of the semiconductor detector on the oscilloscope (coupling = AC, trigger selector = LF) and optimize the position of the semiconductor detector so that the greatest possible output signal is generated.

Note: The distance between two peaks of the output signal corresponds to one half revolution of the mirror, as two mirrors are attached to the opposite sides of the rotating surface.

– Check the quality and position of image S” on the glass scale (S” is fainter when the mirror is rotating).
– You may want to clamp a thin piece of paper between the holder and the glass scale to maximize the contrast of the actual image of the light source and screen out the weaker diffraction phenomena.
– Reduce the output voltage to 0 V. Return the rotary mirror to its initial position using the adjusting wrench, and then remove the wrench.

Carrying out the experiment

– Read position x_0 of image S” from the glass scale and write it down.
– Slowly increase the output voltage of the variable low-voltage transformer (to max. 240 V).
– Read position x of image S” from the glass scale and write it down.
– Determine period T for one complete rotation of the rotary mirror and write this down. Check your result by calculating the motor frequency ν.
– Reduce the output voltage of the variable low-voltage transformer in steps and repeat your measurement for several voltage levels (see Table 1).

Measuring example

Table 1: Motor voltage U, position x of image S”, rotation period T and frequency ν of rotating mirror

$\frac{U}{\text{V}}$	$\frac{x}{\text{mm}}$	$\frac{T}{\text{ms}}$	$\frac{\nu}{\text{Hz}}$
0	10.0 ± 0.3		0
220	15.5 ± 0.3	2.4	417
200	15.0 ± 0.3	2.55	392
150	14.0 ± 0.3	3.2	313
115	13.0 ± 0.3	4.05	247
44	10.5 ± 0.3	26	38.5

Table 1 and Fig. 4 show the measuring example. The error bars drawn in the diagram correspond to a measuring error of 0.3 mm for the image position x. As the motor speed was measured very precisely, it is not necessary to specify the error here.

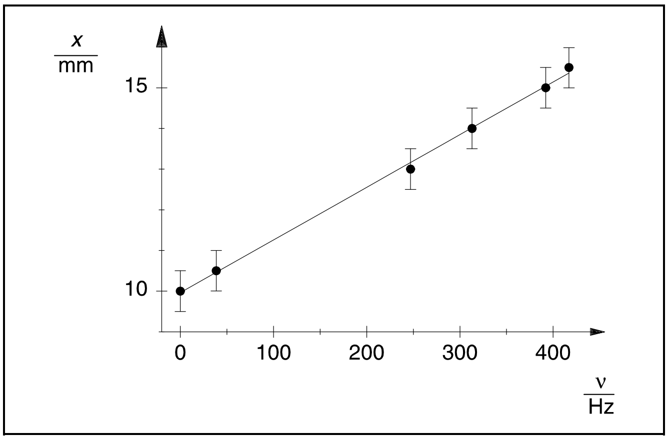


Fig. 4: Position x of image S” on the glass scale as a function of the motor frequency ν.

Evaluation

From Fig. 4, we can obtain the linear relationship between the position x of image S" and the motor frequency ν. The slope is

$$m = 12.9 \cdot 10^{-6}\ \frac{\text{m}}{\text{Hz}}\ .$$

This corresponds to the ratio

$$\frac{\Delta x}{\nu} = \frac{x(\nu) - x(0)}{\nu}$$

in equation (VI). From this, we can calculate the velocity of light as

$$c = 2.84 \cdot 10^{8}\ \frac{\text{m}}{\text{s}}\ .$$

Result

Remark: The straight line drawn in the diagram is the result of a best fit. When calculating the error to obtain a best-fit line, remember that the scattering of the measured values around the line is significantly less than its measuring error. We obtain a value of 7 % as the relative error for the slope of the line. The velocity of light can be specified with the same relative error, as the other quantities from (VI) were measured with more precision.

Experiment result:

$$c = (2.8 \pm 0.2) \cdot 10^{8}\ \frac{\text{m}}{\text{s}}$$

Literature value:

$$c = 3.00 \cdot 10^{8}\ \frac{\text{m}}{\text{s}}$$