Optics

Wave optics

Michelson interferometer

LD Physics Leaflets

P5.3.4.4

Determination of the temporal coherence and line width of spectral lines by means of the Michelson interferometer

Objects of the experiments

- Determination of the wavelength of the green spectral line for an Hg spectral lamp by means of a Michelson interferometer
- Determination of the coherence length of the green spectral line for an Hg spectral lamp by means of a Michelson interferometer
- Determination of the coherence time and line width of the green spectral line

Principles

Coherence is the ability of various waves to create stationary interference effects. A temporally stationary interference structure can only be observed when the phase differences between any partial waves around a fixed point change during the observation time by less than 2π. Then the partial waves are called temporally coherent. The maximum time span Δt, during which the phase differences between all parts of the waves change by a maximum of 2π is called the coherence time.

The coherence time Δt_C is directly associated with the spectral width $\Delta\nu$ of the light source. The following applies:

$\Delta t_C = \frac{1}{\Delta\nu}$. By determination of the coherence time Δt_C the spectral width $\Delta\nu$ or $\Delta\lambda$ of a light source, e.g. of individual spectral lines, is determined from the relationships

$$\Delta\nu = \frac{1}{\Delta t_C} \quad \text{or} \quad \Delta\lambda = \frac{1}{c}\frac{\lambda_0^2}{\Delta t_C}. \qquad \text{(I)}$$

Often the coherence length is used instead of the coherence time. This describes the distance

$$\Delta s_C = \frac{c}{n}\Delta t_C \qquad \text{(II)}$$

the light travels in a medium with a refractive index n during the coherence time.

Natural line widths for spectral lines in the visible spectral range are approx. $\Delta\lambda \approx 10^{-14}\,\text{m}$. This corresponds to a coherence time of approx. 10^{-8} s or a coherence length of 30 m. In commercial spectral lamps the temperature and pressure situation will, however, lead to a noticeable broadening of the spectral lines.

The dominant effect of the line broadening in spectral lamps is pressure broadening. If during the light emission, impact with a further atom occurs, this leads to a change in the photon energy and/or the phase of the emitted wave and therefore to a change in the line width. This effect is linearly associated with the gas pressure and leads to additional shifting of the spectral lines.

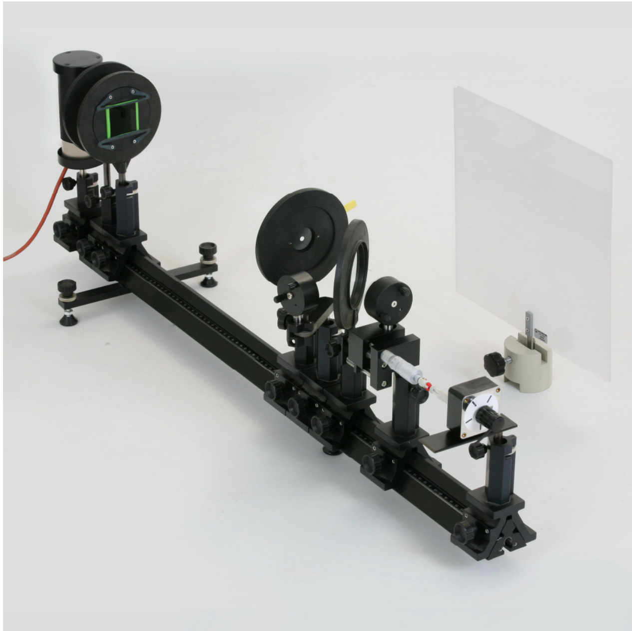

Fig. 1: Experimental setup

A further part of line broadening is based on the Doppler effect because the atoms move statistically in space during the emission. This broadening mechanism increases linearly with the translation speed of the atoms and therefore with increasing temperatures in proportion to $\sqrt{T}$.

In the visible spectral range the Doppler broadening exceeds the natural line width by approximately two orders of magnitude. Typical coherence lengths are several metres for lasers, several centimetres for spectral lines and a few micrometres for white light sources. They can e.g. be measured by means of the Michelson interferometer. The Michelson interferometer is one of the family of two beam interferometers. Interferometer measurements based on this interferometer type are founded on the principle described below:

The beam from a light source is split by a semi-transparent mirror into two parts. The partial beams passing along different paths are reflected back along the same path and are finally recombined and superimposed. The superimposition of the light waves creates the interference image.

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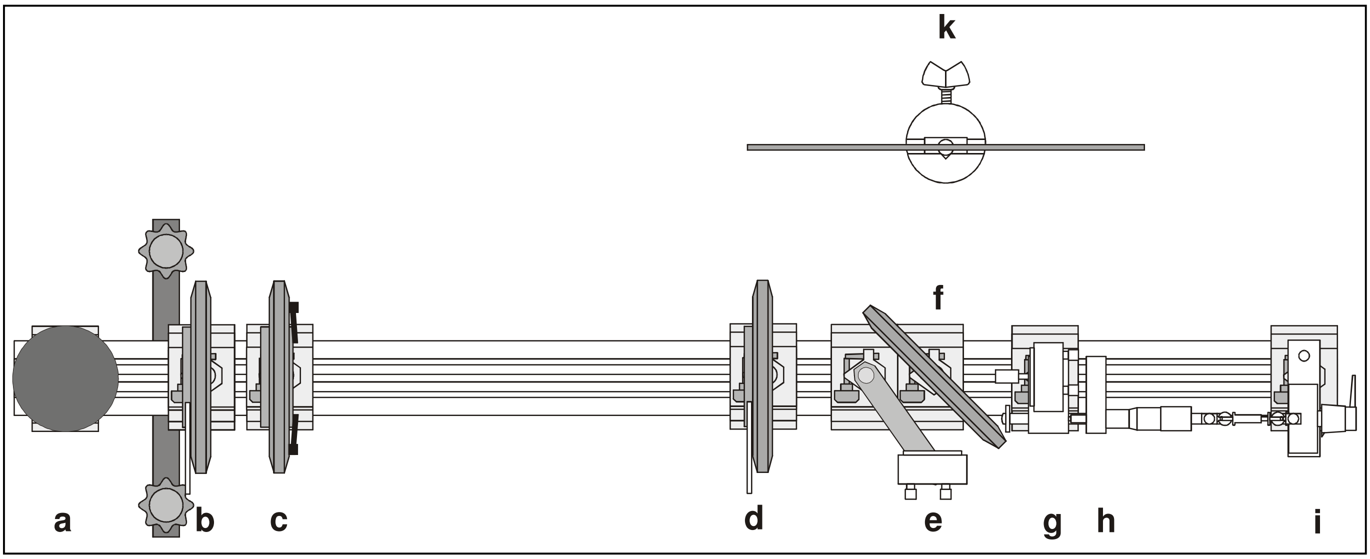

Fig. 2: The setup of the Michelson interferometer on the optical bench seen from above

a spectral lamp Hg 100
b, d iris diaphragm
c monochromatic filter, yellow-green
e, g planar mirror with fine adjustment
f beam splitter
h fine adjustment drive
i reduction gears of the fine adjustment drive
k translucent screen

If the light source emits a slightly divergent beam, the phase differences at different locations on the screen differ. For this reason a homogeneous intensity distribution is not obtained but a system made up from light and dark interference rings, or a system made up from light and dark lines if only a part of the ring system is visible.

In the experiment initially only the wavelength λ of the green spectral line of an Hg spectral lamp is determined by shifting one of the planar mirrors of the Michelson interferometer, by means of the fine adjustment drive, by a precise distance Δs which causes the optical path length of the affected partial beam to be altered. During this shift the interference lines on the observation screen drift. For the evaluation, either the intensity maxima or the intensity minima are counted passing a specific point on the observation screen while the planar mirror is being shifted.

For determination of the coherence lengths, the positions of an adjustable planar mirror are determined for where interference can still just be observed. From the difference in the path lengths the coherence length Δs_C can be directly derived, and from this the coherence time Δt_C and the line width $\Delta \nu$ of the spectral line can be determined.

Apparatus

1 spectral lamp Hg 100 .... 451 062
1 housing for spectral lamps 451 16
1 universal choke 230 V, 50 Hz............................. 451 30

1 optical bench with standard profile, 1 m 460 32
1 optics rider 60/50... 460 373
7 optics riders 90/50 .... 460 374
2 planar mirrors with fine adjustment...................... 473 461
1 fine adjustment drive 473 48
1 cantilever arm.. 460 380
1 beam splitter... 471 88
2 iris diaphragms 460 26
1 monochromatic filter, yellow-green...................... 468 07
1 holder with spring clips 460 22
1 translucent screen 441 53
1 saddle base 300 11

Note:

This experiment can also be carried out for individual lines from other spectral lamps with the appropriate filter. Suitable are e.g. the following combinations:

- *Spectral lamp He (451 031) with dark red (468 01) or yellow (468 05) monochromatic light filter.*
- *Spectral lamp Cd (451 041) with dark red (468 03) monochromatic light filter*

A experimental setup applying the cross connector (460 342) is shown at the end of the leaflet.

Safety notes:

Connect spectral lamp (451 062) in housing (451 16) only via the universal choke (451 30) to the mains.

Between the light opening and the optical element (e.g. diaphragm, lens) a minimum distance of 3 cm has to be adhered to in order to prevent overheating.

Setup

Note: Carry out the measurements in a room as completely dark as possible.

Optical components with damaged or dirty surfaces can lead to errors in the interference pattern.

Treat the planar mirror and the beam splitter with great care, store them protected from dust and never touch with the bare hands.

The experimental setup is shown in figure 2. For setting up, the steps described below are required:

Installation on the optical bench:

- Mount the Hg 100 spectral lamp (**a**) in the optics rider 60/50 at one end of the optical bench.
- Mount the reduction gears for the fine adjustment drive (**i**) with the magnetic base on the gear base and mount it on the other end of the optical bench.
- Clamp a planar mirror (**g**) at the top end of the fine adjustment drive (**h**) and mount it in front of the reduction gears (**i**) on the bench.
- Carefully clamp the universal coupling in the joint head of the micrometer screw of the fine adjustment drive (**h**).
- Shift the optics rider with the fine adjustment drive (**h**) and the height of the gear base of the reduction gears (**i**) in such a way that the coupling rods are neither fully stretched out nor compressed. Otherwise the measurement might become falsified because of shifting of the fine adjustment drive.
- Keep the angle between the individual elements of the couplings as small as possible (and under no circumstance larger than 45°)
- Mount one of the iris diaphragms (**b**) approx. 5 cm behind the spectral lamp Hg 100, the second iris diaphragm (**d**) approx. 25 cm in front of the planar mirror, in such a way that the centres of the diaphragms are at the same height.
- Connect the universal choke to the spectral lamp and switch on; wait for several minutes for the lamp to warm up.

Adjustment of the planar mirror (g)

After the adjustment of the planar mirror (**g**) it should reflect the light from the spectral lamp back onto its own path. Only in this case shifting of the planar mirror (**g**) by means of the fine adjustment drive will not lead to a beam movement.

- Close the iris diaphragms (**b, d**) as much as possible.
- Align the planar mirror (**g**) by adjusting the screws on the back so that the reflected beam hits both iris diaphragm (**d**) and iris diaphragm (**b**) centrally.

After this the adjustment screws on the planar mirror (**g**) must not be touched again! A change to the adjustment of the planar mirror (**g**) would mean that any shifting of the planar mirror (**g**) with the fine adjustment drive would result in a movement of the reflected beam and therefore the partial beams would no longer superimpose properly.

Beam splitter (f) and the second planar mirror (e)

- Place the beam splitter (**f**) as close as possible to the front of the planar mirror (**g**) so that the complete travel of the fine adjustment drive can be utilised. The mirror side of the beam splitter points in the direction of the spectral lamp.
- Turn the beam splitter (**f**) in such a way that the beam reflected from the planar mirror (**g**) is deflected by 90°.
- Fix the translucent screen (**k**) in the base and place it next to the optical bench so that it is hit at its centre.
- Then clamp the second planar mirror (**e**) in the cantilever arm and mount it on the optical bench in such a way that the planar mirror is hit at its centre by the partial beam reflected from the beam splitter. The distances the planar mirror (**e**) and planar mirror (**g**) are from the beam splitter (**f**) should be approximately equal. On account of the relatively small coherence length of the spectral lamp of a few millimetres, the optical wavelength of the two interferometer arms may be only very slightly different. If necessary adjust the positions of the planar mirror (**e**) and/or the beam splitter (**f**).

Superimposition of the partial beams

For the superimposition adjust only the planar mirror (**e**) (on the cantilever arm)!

- Align the planar mirror (**e**) by adjusting the screws on the back so that the beam is nearly reflected along its own path and after transmission through the beam splitter combines with the first partial beam.
- By fine adjustment of the planar mirror (**e**) the beams of the two interferometer arms can be fully superimposed. For doing this it is useful to cover the beam reflected from the planar mirror (**e**) directly in front of the planar mirror (**e**) partially with a piece of firm paper (e.g. a calling card). The position of the partial beam allowed to pass can now easily be compared to the position of the beam reflected by the planar mirror (**g**).
- Fully open iris diaphragm (**d**).

Now interference lines or rings should become visible on the screen (see fig. 3).

- Carefully adjust the planar mirror (**e**) in such a way that on the screen a system of concentric rings becomes visible in the centre of the lit area.
- Open the iris diaphragm (**b**) sufficiently that the contrast in the interference pattern is not affected.

Carrying out the experiment

During the experiment:

- Avoid mechanical vibration of the optical bench (e.g. do not wobble the table).
- Avoid air flow through the setup (because of flow marks) e.g. from draughts.
- Mark a location on the transparent screen (**k**) where the drifting interference lines can be counted.
- Because of the play in the gearbox, adjust the gearbox button slowly and uniformly by gently placing the finger onto the lever of the reduction gears (**i**) and in this way until, if necessary with more turns, the interference lines start moving.
- Then give the gearbox knob at least one further turn before starting with the measurement.

Note: if the planar mirror and therefore the interference pattern moves jerkily, the slide bush of the fine adjustment drive needs to be lubricated.

Determination of the wavelength of the green Hg spectral line

- Clamp the monochrome filter, yellow-green, in the holder with spring clips and mount it behind the iris diaphragm (**b**) on the optical bench.
- Rotate the gearbox knob and at the same time count the interference lines (approx. 100) drifting past and the turn of the reduction gears.

Determination of the coherence length of the green Hg spectral line

- Continue rotating the gearbox knob until the interference pattern is only just visible. Read the setting from the micrometer screw of the fine adjustment drive.
- Rotate the gearbox knob in the other direction to make the interference pattern reappear. Continue rotating until the interference pattern is again only just visible and read the setting from the micrometer screw of the fine adjustment screw.

Measuring examples and evaluation

Determination of the wavelength of the green Hg spectral line

The number N of rotations of the reduction gears, the total shift Δs of the planar mirror, the wavelength λ of the spectral line and the number of rotations Z of the intensity maxima are given by the relationship below:

$Z \cdot \lambda = 2 \cdot \Delta s$ with $\Delta s = N \cdot 5$ µm

The factor 2 in this equation takes into account that the geometric path is changed for both the direct and the reflected beam by Δs.

Therefore for λ

$$\lambda = 2 \cdot \frac{\Delta s}{Z} = 2 \cdot \frac{N \cdot 5 \mu m}{Z}$$

In an example, the number of intensity maxima counted was Z = 73 and the number of turns N = 4. From this the wavelength for the green spectral line of the Hg spectral lamp is determined: λ = 548 nm.

The value given in the literature is λ_{theo} = 546.01 nm.

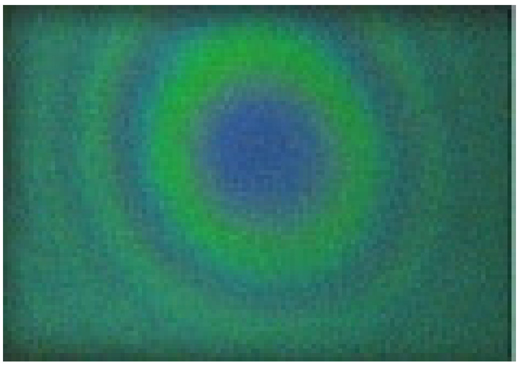

Fig. 3: Interference pattern on the screen without filter

Determination of the coherence length of the green Hg spectral line

The coherence length Δs_C corresponds to the difference between the two optical path lengths and therefore the two mirror positions x_1 and x_2 where interference can still just be observed:

$\Delta s_C = 2\ (x_2 - x_1)$

The example resulted in x_1 = 12.5 mm, x_2 = 18.7 mm and therefore Δs_C = 12.4 mm.

Calculation of the coherence time and line width of the green spectral line

The coherence time results from formula (II)

$$\Delta t_C = \frac{n}{c} \Delta s_C .$$

With Δs_C = 12.4 mm one obtains with n = 1.0 and $c = 3.0 \cdot 10^8 \frac{m}{s}$ for the coherence time $\Delta t_C = 4.1 \cdot 10^{-11}$ s.

The line width of the green Hg spectral line is found from the formula (I): $\Delta \nu = \frac{1}{\Delta t_C}$ or $\Delta \lambda = \frac{1}{c} \frac{\lambda_0^2}{\Delta t_C}$.

For $\Delta t_C = 4.1 \cdot 10^{-11}$ s and λ = 548 nm this gives $\Delta \nu = 2.4 \cdot 10^{10} \frac{1}{s}$ or $\Delta \lambda = 2.4 \cdot 10^{-11}$ m .

Discussion

By means of the interferometric measurement the wavelength of the green spectral line of the Hg spectral lamp can be determined precisely. The measuring precision is the higher the larger the total shift Δs and therefore the number of turns and the number of counted intensity maxima.

The observed coherence length of 12.5 mm shows, as described in Principles, that the temperature and pressure situation inside the Hg spectral lamp is sufficient to lead to a marked broadening of the spectral line.

The influence of different pressure and temperature conditions on the coherence length is therefore investigated in more detail in experiment P5.3.4.5.

Notes:

If the monochromatic filter, yellow-green, is removed from the path of the beam and the planar mirror (**g**) is shifted carefully by rotating the gear button, it is apparent that in certain regions interference images are obtained with a strong light-dark contrast while in other positions a coloured ring system is obtained.

In the case of double lines the superposition in the Michelson interferometer leads to further variations in intensity. This effect can be utilised in order to determine the line separation in the case of double lines (see experiment P5.3.4.6).

Alternative Setup with cross connector (460 342)

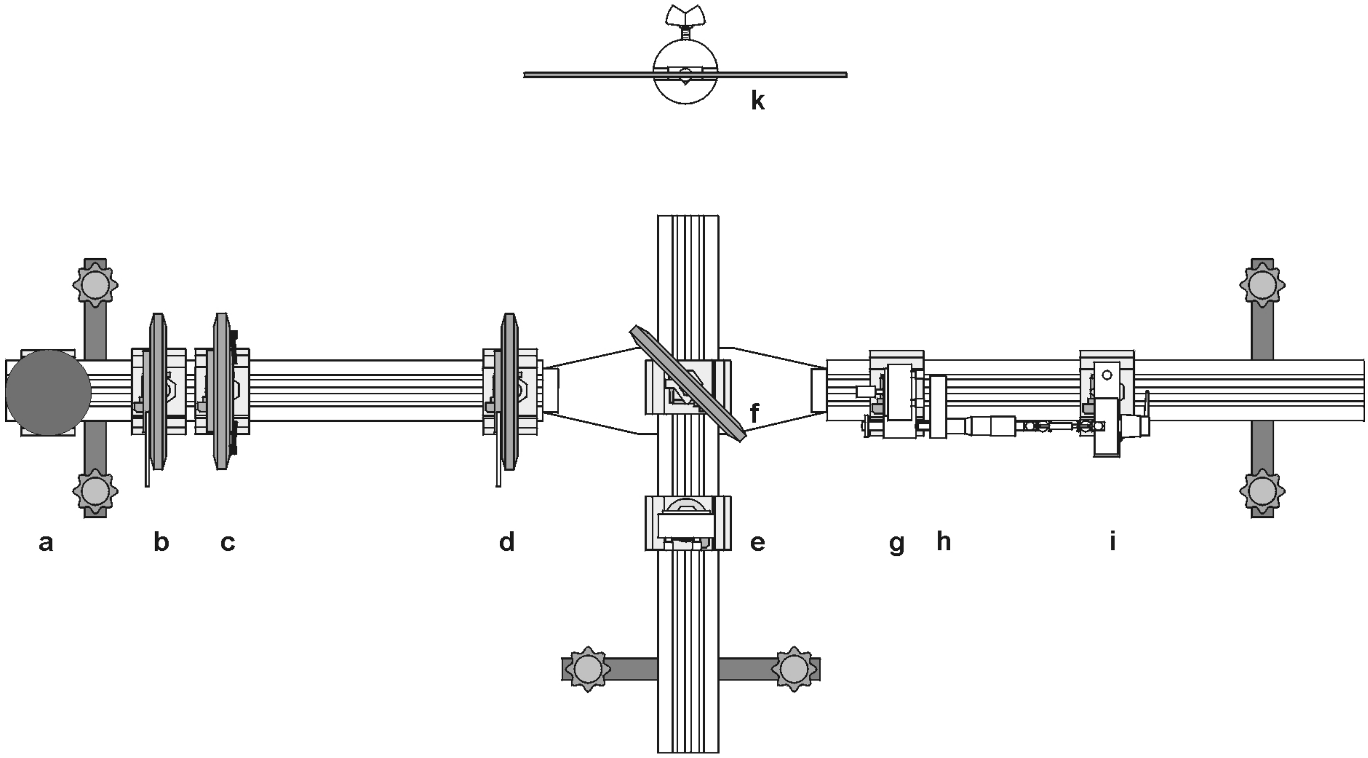

Fig. 4: The setup of the Michelson interferometer on the optical bench seen from above

- **a** spectral lamp Hg 100
- **b, d** iris diaphragm
- **c** monochromatic filter, yellow-green
- **e** planar mirror with fine adjustment in extension rod
- **f** beam splitter
- **g** planar mirror with fine adjustment
- **h** fine adjustment drive
- **i** reduction gears of the fine adjustment drive
- **k** translucent screen

Connect the optical benches with the cross connector as shown in Fig. 4. Then, the setup of the optical components is similar to the setup on a single optical bench.

Apparatus

1 spectral lamp Hg 100 .... 451 062
1 housing for spectral lamps 451 16
1 universal choke 230 V, 50 Hz.............................. 451 30

3 optical bench with standard profile, 0.5 m 460 335
1 cross connector.. 460 342
1 optics rider 60/50.. 460 373
7 optics riders 90/50.. 460 374
1 extension rod.. 460 385
2 planar mirrors with fine adjustment 473 461
1 fine adjustment drive 473 48
1 beam splitter... 471 88
2 iris diaphragms... 460 26
1 monochromatic filter, yellow-green...................... 468 07
1 holder with spring clips....................................... 460 22

1 translucent screen.. 441 53
1 saddle base.. 300 11