Electronics

Components, basic circuits
Current and voltage sources

Determining
the internal resistance
of a battery

Objects of the experiment

■ Measuring the terminal voltage U as a function of the current I flowing through a load resistor.

■ Determining the internal resistance R_i, the open-circuit voltage U_0 and the short-circuit current I_S.

■ Calculating the power P supplied as a function of the load resistance R.

■ Determining the maximum power P_{max} and the associated values R_{max}, U_{max} and I_{max} for the load resistance, the open-circuit voltage and the current drawn.

Principles

The open-circuit voltage U_0 generated in a voltage source is, in general, different from the terminal voltage U measured at the terminals when a current I is drawn from the voltage source. When the maximum current that can be drawn from the voltage source, the short-circuit current I_S, flows, the terminal voltage breaks down to become zero. That means, there must be an internal resistance R_i acting in the voltage source, where part of the voltage generated drops. This resistance is called the internal resistance of the voltage source.

In the experiment, a rheostat is connected to a battery as an ohmic load for determining the internal resistance. The terminal voltage U of the battery is measured for various loads and plotted against the current I flowing through the load resistor. Under the assumption that the internal resistance R_i is constant, a straight line is fitted to the measured values according to

$$U = U_0 - R_i \cdot I \qquad (I).$$

From the slope of the straight line, the internal resistance is determined. As R_i is constant, the short-circuit current is

$$I_S = \frac{U_0}{R_i} \qquad (II).$$

In a second diagram, the power supplied,

$$P = U \cdot I \qquad (III),$$

is plotted as a function of the load resistance

$$R = \frac{U}{I} \qquad (IV).$$

Because of (I), (III) and (IV), the power fulfils the following relation:

$$P = U_0^2 \cdot \frac{R}{\left(R + R_i\right)^2} \qquad (V).$$

The power P supplied attains its maximum when the load resistance R is equal to the internal resistance R_i, and the maximum power is

$$P_{max} = \frac{U_0^2}{4 \cdot R_i} = \frac{1}{4} \cdot U_0 \cdot I_S \qquad (VI).$$

At maximum power the terminal voltage corresponds to half the open-circuit voltage, i.e.

$$U_{max} = \frac{U_0}{2} \qquad (VII),$$

and the current drawn is half the short-circuit current, i.e.

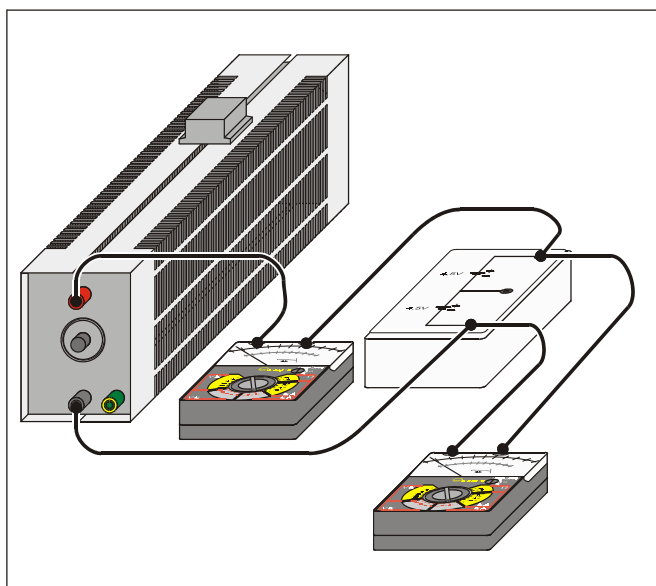$$I_{max} = \frac{U_0}{2 \cdot R_i} = \frac{I_S}{2} \qquad (VIII).$$

Fig. 1   Experimental setup for determining the internal resistance of a battery

Apparatus

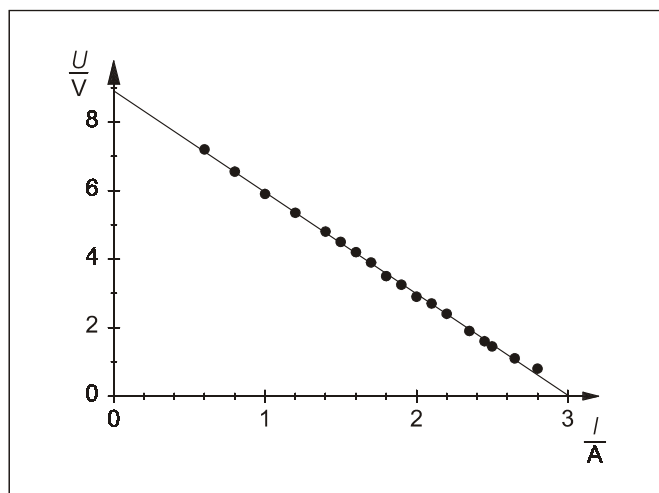|	
1 Battery case, 2 × 4.5 V	576 89
1 set of 20 monocells, 1.5 V	503 11
1 voltmeter, DC, $U \le 10$ V	e. g. 531 120
1 ammeter, DC, $I \le 3$ A	e. g. 531 120
1 rheostat 10 Ω, 8 A	537 32
Connecting leads	

Setup

The experimental setup is illustrated in Fig. 1.

– Put six monocells into the battery case so that they are series connected.

– Connect the battery case, the ammeter and the load resistor in series; however, do not close the circuit yet to spare the batteries.

– Connect the voltmeter to the sockets of the battery case.

Carrying out the experiment

– Close the circuit, and adjust the rheostat so that the smallest possible current flows.

– Read the current and the voltage, and take them down.

– Changing the load resistance, increase the current step by step, but quickly, up to 3 A at maximum, each time reading and taking down the current and the voltage.

– Having finished the series of measurements, interrupt the circuit.

Measuring example

Table 1: Measured values of the terminal voltage U of the battery and the current I flowing through the load resistor

$\dfrac{U}{V}$	$\dfrac{I}{A}$
7.20	0.60
6.55	0.80
5.90	1.00
5.35	1.20
4.80	1.40
4.50	1.50
4.20	1.60
3.90	1.70
3.50	1.80
3.25	1.90
2.90	2.00
2.70	2.10
2.40	2.20
1.90	2.35
1.60	2.45
1.45	2.50
1.10	2.65
0.80	2.80

Fig. 2   Terminal voltage U as a function of the current I.

Evaluation

a) Terminal voltage U as a function of the current I:

In Fig. 2, the measured terminal voltage U is shown graphically as a function of the current I. Within the accuracy of measurement, the measured values lie on the straight line drawn in the diagram having a slope of 2.96 V A^{-1} and an intercept on the U-axis of 8.91 V.

Hence, the internal resistance of the battery is $R_i = 2.96$ Ω and the open-circuit voltage is $U_0 = 8.91$ V. With the aid of Eq. (II), the short-circuit current is calculated: $I_S = 3.01$ A.

The entire battery is a series connection of six equal monocells. Their open-circuit voltage is

$$U_0' = \frac{1}{6} \cdot U_0 = 1.49 \text{ V}$$

and their internal resistance is

$$R_i' = \frac{1}{6} \cdot R_i = 0.49 \text{ Ω} .$$

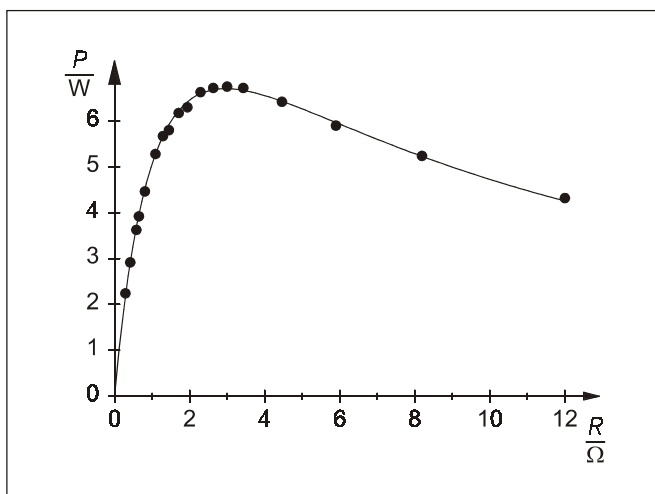**b) Power *P* supplied as a function of the load resistance *R*:**

Table 2: measured values of *U* and *I* and calculated values of *P* and *R*

$\dfrac{U}{V}$	$\dfrac{I}{A}$	$\dfrac{P}{W}$	$\dfrac{R}{\Omega}$
7.20	0.60	4.32	12.00
6.55	0.80	5.24	8.19
5.90	1.00	5.90	5.90
5.35	1.20	6.42	4.46
4.80	1.40	6.72	3.43
4.50	1.50	6.75	3.00
4.20	1.60	6.72	2.63
3.90	1.70	6.63	2.29
3.50	1.80	6.30	1.94
3.25	1.90	6.18	1.71
2.90	2.00	5.80	1.45
2.70	2.10	5.67	1.29
2.40	2.20	5.28	1.09
1.90	2.35	4.47	0.81
1.60	2.45	3.92	0.65
1.45	2.50	3.63	0.58
1.10	2.65	2.92	0.42
0.80	2.80	2.24	0.29

In Table 2, the power *P* supplied and the load resistance *R* calculated from the measured values of *U* and *I* of Table 1 according to Eqs. (II) and (III) are listed. The maximum power P_{max} = 6.75 W is supplied to the load resistance at R_{max} = 3.0 Ω, U_{max} = 4.5 V or I_{max} = 1.5 A, respectively.

Fig. 3 shows a diagram of the values from the table. Within the accuracy of measurement, they lie on the curve calculated with the aid of Eq. (V) with the parameters R_i = 2.96 Ω und U_0 = 8.91 V. With these parameters, Eqs. (VI), (VII) and (VIII) give: P_{max} = 6.71 W, U_{max} = 4.46 V and I_{max} = 1.51 A.

Fig. 3   Power *P* supplied as a function of the load resistance *R*.

Result

In a circuit, a real battery behaves like a series connection of a constant voltage source and an ohmic resistor. Therefore the maximum current that can be drawn from the battery is limited by the corresponding resistance.

The maximum power is supplied to the load resistor when the load resistance is equal to the internal resistance. In this case, the same power is converted "at the internal resistance". The current then is half the short-circuit current, and the terminal voltage is half the open-circuit voltage.

Supplementary information

The voltage of a new, unloaded monocell (Leclanché battery) is approximately 1.56 V. The extrapolation of the measured values described renders a somewhat smaller open-circuit voltage because the electrolytic solution inside a Leclanché battery becomes alkaline while it supplies current due to the consumption of H_3O^+ ions. After current has been drawn, OH^- ions diffuse back into the electrolyte. Therefore the open-circuit voltage increases gradually (recovery of the Leclanché battery).

It is not possible to record the series of measurements described here with chargeable accumulators because these have an internal resistance of only a few mΩ.