Determining the impedance in circuits with capacitors and coils

Aim of the test

- Determine the resonance frequency with circuits of capacitors and coils in series and parallel
- Determine the AC resistance depending on the frequency with circuits of capacitors and coils in series and parallel
- Observe the phase shift

Foundations

In order to examine the AC resistance a circuit is constructed with a coil (inductance *L*), a capacitor (capacitance *C*) and a resistor *R*. This type of circuit is also known as an RCL resonant circuit. If a charged capacitor is discharged within it via a coil then the voltage at the capacitor does not drop to zero exponentially but instead oscillates. The energy is thereby transferred between electrical and magnetic fields. The Thomson equation applies to the resonant frequency f_R:

$$f_R = \frac{1}{2\pi\sqrt{LC}} \qquad (1)$$

With *L* : Inductance, *C*: Capacitance

In the test the resonant circuit is tested with the capacitance and inductance in series and in parallel. For this purpose the voltage drop is measured across a resistor with an oscilloscope and thereby the current strength dependent on the frequency applied.

The following applies to the AC resistance of the LC part

when wired in series $$Z_S = \left| 2\pi f L - \frac{1}{2\pi f C} \right| \qquad (2)$$

and

in parallel: $$\frac{1}{Z_P} = \left| \frac{1}{2\pi f L} - 2\pi f C \right| \qquad (3)$$

By applying the resonant frequency f_R the right side of the equation becomes zero, i.e. the resistance disappears in the series circuit and becomes infinite in the parallel circuit.

The series circuit is known as a filter circuit or a band-pass filter. Only a certain frequency band of the input signal is able to pass the filter. Frequencies outside the resonant frequency are attenuated accordingly.

The parallel circuit is used as a trap circuit or bandstop. Resonant frequencies are blocked. All other frequencies are able to pass the filter.

For the phase shift φ with RCL series connection (see Fig. 1), the following applies:

$$\tan\varphi = \frac{2\pi f L - \frac{1}{2\pi f C}}{R} \qquad (4)$$

In resonant frequency the phase shift is 0°. With very low frequencies it runs to –90°, and with very high frequencies to +90°.

With parallel connection the following applies for the phase shift φ:
(see Fig. 2):

$$\tan\varphi = \frac{1}{R\left(\frac{1}{2\pi f L} - 2\pi f C\right)} \qquad (5)$$

With very low frequencies and with very high frequencies the phase shift runs to 0°. When approaching the resonant frequency it runs to + or – 90°. If the resonant frequency is exceeded then the phase jumps 180°.

Devices

1 plug-in board, DIN A 4	576 74
1 STE resistor 10 Ω	577 20
1 STE resistor 100 Ω	577 32
1 STE capacitor 1 µF	578 15
1 STE capacitor 4.7 µF	578 16
1 coil, 500 windings	590 83
1 coil, 1000 windings	590 84
1 function generator S 12	522 621
1dual channel oscilloscope 303	575 211
2 measuring cable BNC/4 mm	575 24
1 pair of cables, red and blue, 100 cm	501 46

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Apparatus and method

a) Determining the resonant frequency

Series connection

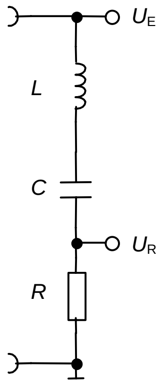


Fig. 1

– Test apparatus per Figure 1. LC combination per Table 1. Resistor R = 100 Ω.

Attention! It is essential to use one resistor in the circuit because – due to the resonance rises at the coil and capacitor in series circuits – considerably higher voltages than the input voltage may arise.

– First set the frequency generator to 100 Hz (Sinus) and set a peak voltage of U_E = 6 V with the aid of the oscilloscope.

– During the experiment the timebase of the oscilloscope must be aligned with the respective frequency setting.

– Increase the frequency until U_R is maximum (i.e. Z minimum)

– Enter the frequency in Table 1.

– Measure using further LC combinations in accordance with Table 1.

Parallel connection

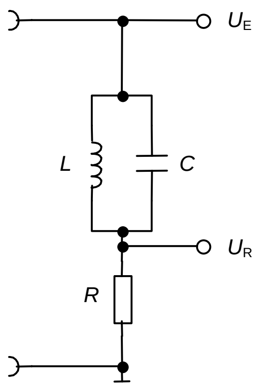


Fig. 2

– Test apparatus per Figure 2. LC combination per Table 2. Resistor R = 100 Ω.

– Increase the frequency until U_R is minimum (i.e. Z maximum)

– Enter the frequency in Table 2.

– Measure using further LC combinations in accordance with Table 2.

b) Determining the AC resistance

Series connection

– First rearrange the test apparatus into series connection with the coil 1000 windings, the capacitor 1 μF and the resistor 100 Ω.

– Increase the frequency from 50 Hz to 20,000 Hz in stages. Enter frequencies f and peak voltages U_R in Table 3.

– Repeat the measurement with the resistor 10 Ω.

Parallel connection

– Repeat the test with the circuit in parallel connection and enter the measured values in Table 4.

c) Observing the phase relation

Series connection

– First rearrange the test apparatus into series connection with the coil 1000 windings, the capacitor 1 μF and the resistor 100 Ω.

– Set the resonant frequency and observe the phase.

– Increase the frequency slightly and then decrease once more. When doing so observe the phase relation.

– Set a low frequency (approx. 100 Hz) followed by a high frequency and observe the phase relation. If necessary increase the amplification of the output signal.

Parallel connection

– Repeat the test with parallel connection.

Measurement examples

a) Determining the resonant frequency

Tab. 1: Resonant frequencies in series connection

L \ C	1 μF	4.7 μF
500 windings	2222 Hz	1064 Hz
1000 windings	1220 Hz	568 Hz

Tab. 2: Resonant frequencies in parallel connection

L \ C	1 μF	4.7 μF
500 windings	2182 Hz	1042 Hz
1000 windings	1200 Hz	600 Hz

b) Determining the AC resistance

Tab. 3: Series connection

	R = 100 Ω	R = 10 Ω
$\frac{f}{\text{Hz}}$	$\frac{U_R}{\text{V}}$	
50	0.20	0.020
200	0.8	0.08
500	2.0	0.22
1000	4.3	1.1
1220	5.0	2.0
1500	4.5	0.9
2000	3.5	0.44
5000	1.2	0.13
10000	0.54	0.056
20000	0.27	0.027

Tab. 4: Parallel connection

	R = 100 Ω	R = 10 Ω
$\frac{f}{\text{Hz}}$	$\frac{U_R}{\text{V}}$	
50	5.0	2.2
200	5.0	1.8
500	4.4	0.9
1000	1.7	0.21
1220	0.3	0.09
1500	1.8	0.22
2000	3.6	0.46
5000	5.8	1.8
10000	6.0	3.2
20000	6.0	4.4

Evaluation

The following applies to the inductance of a coil:

$$L = \mu_0 \, N^2 \, \frac{A}{l}$$

with $\mu_0 = 1{,}26 \cdot 10^{-6} \, \frac{\text{H}}{\text{m}}$; N: no. of windings and

$\frac{A}{l} \approx 1{,}4 \cdot 10^{-3} \, \text{m}$ with the coil formers used.

This results in:

$L_{500\,\text{Wdg}} \approx 4{,}4 \, \text{mH}$ $\qquad$ $L_{1000\,\text{Wdg}} \approx 17{,}6 \, \text{mH}$.

a) Calculating the resonant frequency

Tab. 5: Resonant frequencies calculated in accordance with (1)

$f_R = \frac{1}{2\pi\sqrt{LC}}$	1 µF	4.7 µF
4.4 mH	2399 Hz	1107 Hz
17.6 mH	1200 Hz	553 Hz

The calculated frequencies concur with the measured values with both series and parallel connections.

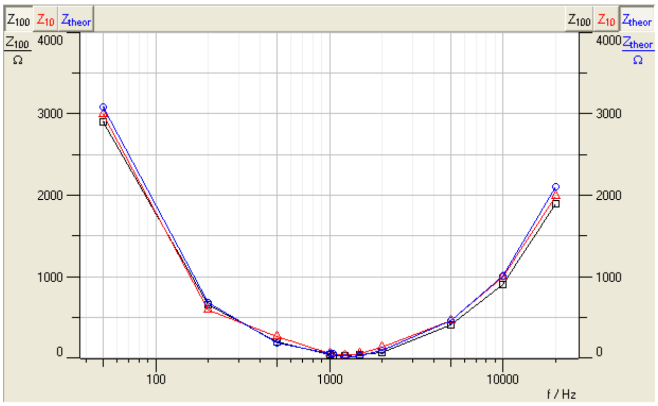
b) Determining the AC resistance

– Calculate the AC resistance of the series circuit with the aid of the measured voltage and in accordance with the formula (2) and enter in a graph.

Tab. 6: Series connection

	R = 100 Ω	R = 10 Ω	
f	$Z = \frac{(U_E - U_R) \cdot R}{U_R}$		Z_{theor} in acc. with (2)
Hz	Ω		Ω
50	2900	2990	3080
200	650	740	680
500	186	263	180
1 000	40	45	50
1 220	20	20	0
1 500	33	56	30
2 000	71	126	90
5 000	400	452	460
10 000	1011	1061	1010
20 000	2122	2212	2110

Diagram 1 (logarithmic graph):
AC resistance of the LC series circuit



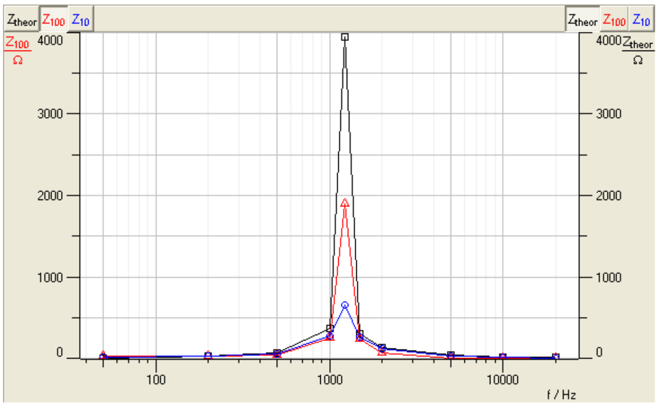
The AC resistances measured with the series circuit clearly concur with the calculated Z_{theor}. The deviation with the resonant frequency in particular can be explained by the additional ohmic resistance of the coil.

– Calculate the AC resistance of the parallel circuit with the aid of the measured voltage and in accordance with the formula (3) and enter in a graph.

Tab. 7: Parallel connection

	R = 100 Ω	R = 10 Ω	
f	$Z = \frac{(U_E - U_R) \cdot R}{U_R}$		Z_{theor} in acc. with (3)
Hz	Ω		Ω
50	20	17	6
200	20	23	23
500	36	57	67
1 000	253	276	362
1 220	1900	657	3950
1 500	233	263	295
2 000	67	120	125
5 000	3	23	34
10 000	0	9	16
20 000	0	4	8

Diagram 2 (logarithmic graph):
AC resistance of the LC parallel circuit



The AC resistances measured concur with the calculated Z_{theor}. With resonant frequency high frequency components of the non-precise sinusoidal input voltage are effective, so that corresponding voltage portions are measured. A further part of the deviation can be attributed to the additional ohmic resistance of the coil.

c) Observing the phase relation

Series connection

With resonant frequency the phase relation is equal, i.e. $\varphi = 0°$ applies to the phase shift.

The phase relation shifts accordingly with changes in the frequency to the right (+) or the left (-).

Parallel connection

With resonant frequency no phase can be observed.
With changes in the frequency a phase shift of 90° (frequency somewhat below the resonant frequency) or –90° (frequency somewhat above the resonant frequency) is evident.