Determining the impedance in circuits with capacitors and ohmic resistors

Objects of the experiment

- Determining the total impedance and the phase shift in a series connection of a capacitor and a resistor.
- Determining the total impedance and the phase shift in a parallel connection of a capacitor and a resistor.

Principles

If an alternating voltage

$$U = U_0 \cdot \cos(\omega \cdot t) \text{ with } \omega = 2\pi \cdot f \quad \text{(I)}$$

is applied to a capacitor with the capacitance *C*, the current flowing through the capacitor is

$$I = U_0 \cdot \omega \cdot C \cdot \cos\left(\omega \cdot t + \frac{\pi}{2}\right) \quad \text{(II).}$$

Therefore a capacitive reactance

$$X_C = \frac{1}{\omega \cdot C} \quad \text{(III)}$$

is assigned to the capacitor, and the current is said to be phase-shifted with respect to the voltage by 90° (see Fig. 1). The phase shift is often represented in a vector diagram.

Series connection

If the capacitor in connected in series with an ohmic resistor, the same current flows through both components. This current can be written in the form

$$I = I_0 \cdot \cos(\omega \cdot t + \varphi_S) \quad \text{(IV)}$$

where φ_S is unknown for the time being. Correspondingly, the voltage drop is

$$U_R = R \cdot I_0 \cdot \cos(\omega \cdot t + \varphi_S) \quad \text{(V)}$$

at the resistor and

$$U_C = X_C \cdot I_0 \cdot \cos\left(\omega \cdot t + \varphi_S - \frac{\pi}{2}\right) \quad \text{(VI)}$$

at the capacitor. The sum of these two voltages is

$$U_S = \sqrt{R^2 + X_C^2} \cdot I_0 \cdot \cos(\omega \cdot t) \quad \text{(VII)}$$

if φ_S fulfils the condition

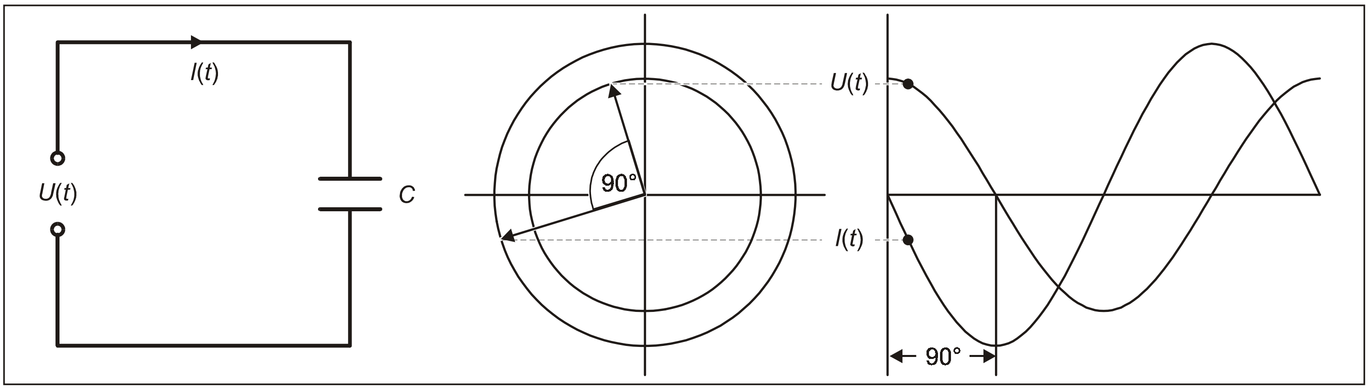
$$\tan \varphi_S = \frac{X_C}{R} \quad \text{(VIII).}$$

U_S is equal to the voltage *U* applied, and therefore

$$U_0 = \sqrt{R^2 + X_C^2} \cdot I_0 \quad \text{(IX),}$$

i.e. the series connection of an ohmic resistor and a capacitor can be assigned the impedance

Fig. 1 AC circuit with a capacitor (circuit diagram, vector diagram and *U*(*t*),*I*(*t*) diagram)



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Apparatus

1 plug-in board A4	576 74
1 resistor 1 Ω, 2 W, STE 2/19	577 19
1 resistor 100 Ω, 2 W, STE 2/19	577 32
1 capacitor 0.1 µF, 100 V, STE 2/19	578 31
1 capacitor 1 µF, 100 V, STE 2/19	578 15
1 capacitor 10 µF, 100 V, STE 2/19	578 12
1 function generator S 12	522 621
1 two-channel oscilloscope 303	575 211
2 screened cables BNC/4 mm	575 24
Connecting leads	

$$Z_S = \sqrt{R^2 + X_C^2} \qquad \text{(X).}$$

In this arrangement, the current is phase-shifted with respect to the voltage by the angle φ_S (see Fig. 2).

Parallel connection

If the capacitor is connected in parallel to the ohmic resistor, the same voltage is applied to both of them. The voltage has, for example, the shape given in Eq. (I). The current flowing through the ohmic resistor is

$$I_R = \frac{U_0}{R} \cdot \cos(\omega \cdot t) \qquad \text{(XI)}$$

whereas the current flowing through the capacitor is

$$I_C = \frac{U_0}{X_C} \cdot \cos\left(\omega \cdot t + \frac{\pi}{2}\right) \qquad \text{(XII).}$$

The sum of the two currents is

$$I_P = \sqrt{\frac{1}{R^2} + \frac{1}{X_C^2}} \cdot U_0 \cdot \cos(\omega \cdot t + \varphi_P) \qquad \text{(XIII)}$$

with

$$\tan\varphi_P = \frac{R}{X_C} \qquad \text{(XIV).}$$

It corresponds to the total current drawn from the voltage source. Hence, the parallel connection of an ohmic resistor and a capacitor can be assigned an impedance Z_P, for which the relation

$$\frac{1}{Z_P} = \sqrt{\frac{1}{R^2} + \frac{1}{X_C^2}} \qquad \text{(XV)}$$

holds. In this arrangement, the current is phase-shifted by the angle φ_P with respect to the voltage (see Fig. 3).

In the experiment, the current $I(t)$ and the voltage $U(t)$ are measured as time-dependent quantities in an AC circuit by means of a two-channel oscilloscope. A function generator is used as a voltage source with variable amplitude U_0 and variable frequency f. From the measured quantities the magnitude of the total impedance Z and the phase shift φ between the current and the voltage are determined.

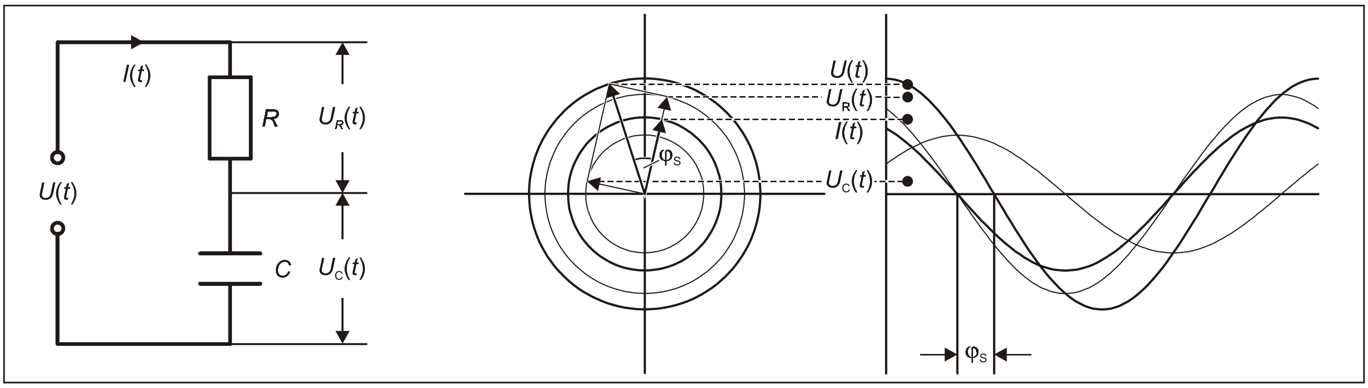


Fig. 2 AC circuit with a capacitor and an ohmic resistor in series connection (circuit diagram, vector diagram and $U(t)$,$I(t)$ diagram)

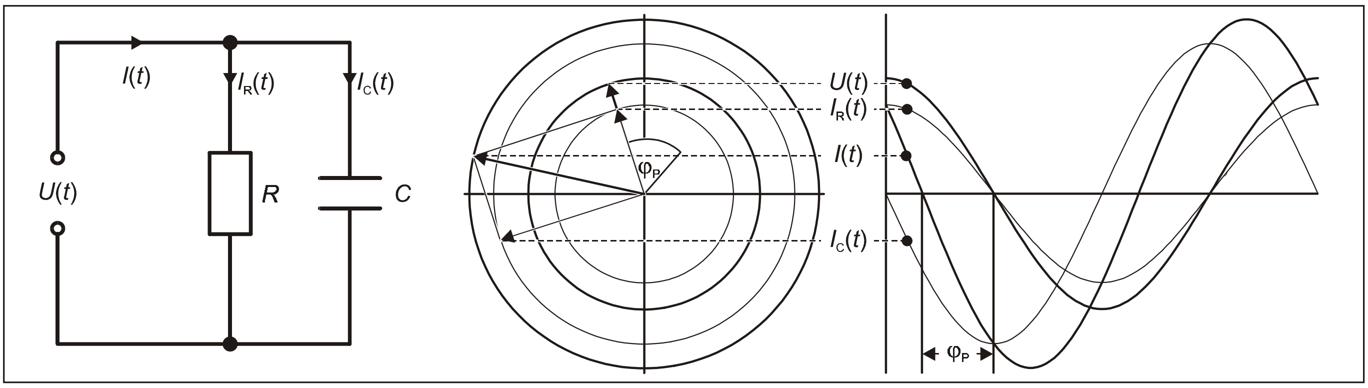


Fig. 3 AC circuit with a capacitor and an ohmic resistor in parallel connection (circuit diagram, vector diagram and $U(t)$,$I(t)$ diagram)

Setup

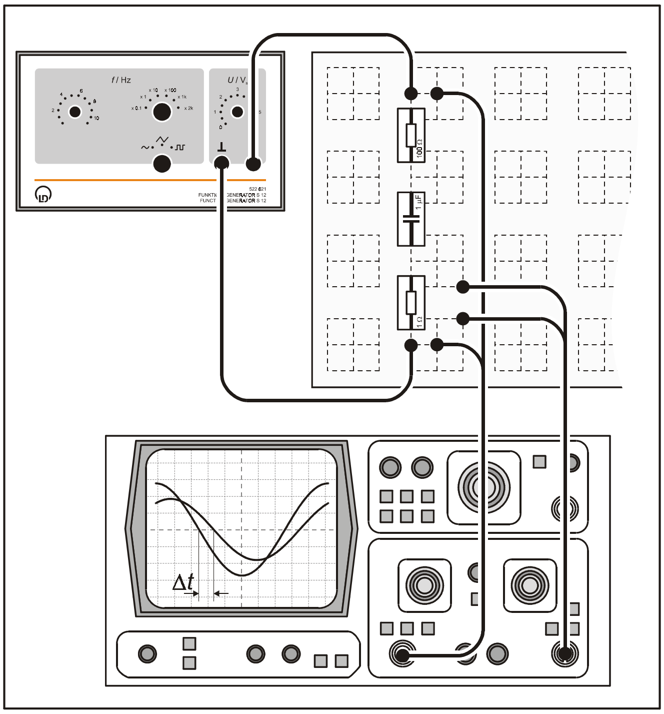
The experimental setup is illustrated in Fig. 4.

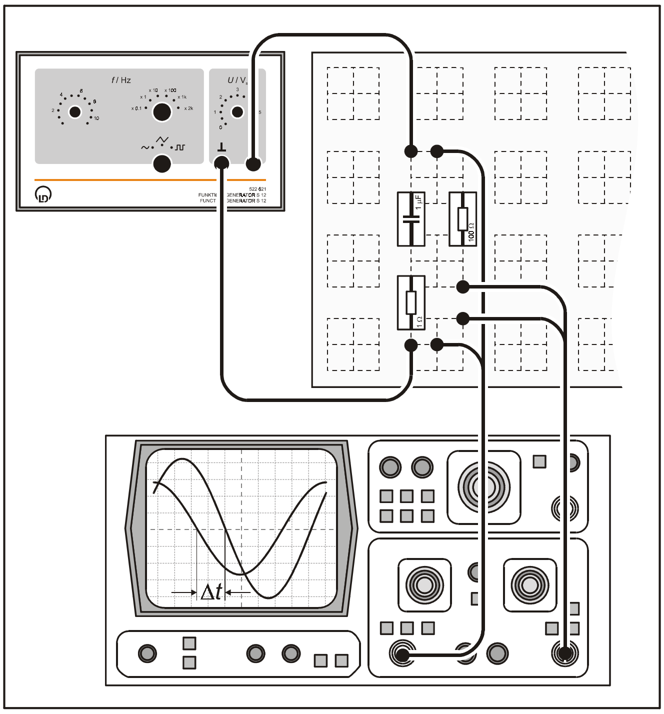
– Connect the function generator as an AC voltage source, and select the shape ∿.

– Connect the channel I of the oscilloscope to the output of the function generator, and feed the voltage drop at the measuring resistor into the channel II.

– Press the DUAL pushbutton at the oscilloscope, and select AC for the coupling and the trigger.

Fig. 4 Experimental setup for determining the impedance in circuits with capacitors and ohmic resistors in series connection (above) and in parallel connection (below)





Carrying out the experiment

– Connect the 10 μF capacitor as a capacitance in series with the 100 Ω resistor.

– Switch the function generator on by plugging in the plug-in power supply, and adjust a frequency of 2000 Hz (T = 0.5 ms).

– Select an appropriate time-base sweep at the oscilloscope.

– Adjust an output signal of 5 V.

– Read the amplitude of the signal in the channel II of the oscilloscope, and enter it in the table as current $I_0 = \frac{U_m}{1\Omega}$.

– Read the time difference Δt between the zero passages of the two signals.

– Connect the 10 μF capacitor in parallel to the 100 Ω resistor.

– Repeat the measurement.

– Repeat the measurements with the 1 μF capacitor and with the 0.1 μF capacitor.

– Adjust other frequencies according to Table 1, and repeat the measurements.

Measuring example

U_0 = 5.0 V, R_m = 1 Ω, R = 100 Ω

Table 1: measuring data for the frequency f, oscillation period T, capacitance C, time difference Δt and current amplitude I_0

			Series connection		Parallel connection	
$\frac{f}{\text{Hz}}$	$\frac{T}{\text{ms}}$	$\frac{C}{\mu\text{F}}$	$\frac{I_0}{\text{mA}}$	$\frac{\Delta t}{\text{ms}}$	$\frac{I_0}{\text{mA}}$	$\frac{\Delta t}{\text{ms}}$
2000	0.5	10	48	0.01	620	0.11
		1	38	0.06	82.5	0.07
		0.1	6	0.12	50	0.01
1000	1	10	48	0.03	330	0.21
		1	27	0.16	58	0.09
		0.1	3.5	0.23	48	0.01
500	2	10	46	0.10	170	0.38
		1	15	0.4	51	0.11
200	5	10	40	0.55	80	0.70
		1	6	1.1	48	0.10
100	10	10	26	1.6	60	0.90
		1	3.5	2.4	50	0.10
50	20	10	15	3.8	52	1.0

Evaluation

The measuring data of Table 1 are evaluated as follows:

The phase shift φ is calculated from the time difference Δt between the voltage and the current and from the oscillation period T according to

$$\varphi = 360 \cdot \frac{\Delta t}{T}$$

and the magnitude of the total impedance is obtained from the amplitudes U_0 and I_0 according to

$$Z = \frac{U_0}{I_0} .$$

The results are listed in Table 2, where the capacitive reactance of the respective capacitor calculated according to Eq. (III) is also given.

For the series connection, Fig. 5 shows a plot of the impedance Z_S and Fig. 6 shows the phase shift φ_S between the current and the voltage, both as functions of the capacitive reactance X_C. The solid lines were calculated according to Eq. (X) and Eq. (VIII), respectively.

The corresponding diagrams for the parallel connection are shown in Figs. 7 and 8. In this case, the solid curves are obtained according to Eqs. (XIV) and (XV).

Table 2: values of the total impedance Z and the phase shift φ between the current and the voltage calculated from the measuring data from Table 1

			Series connection		Parallel connection	
$\frac{f}{\text{Hz}}$	$\frac{C}{\mu\text{F}}$	$\frac{X_C}{\Omega}$	$\frac{Z}{\Omega}$	φ	$\frac{Z}{\Omega}$	φ
2000	10	8.0	104	7°	8.1	79°
	1	80	132	40°	61	50°
	0.1	800	830	86°	100	7°
1000	10	16.0	104	11°	15.2	76°
	1	160	185	58°	86	32°
	0.1	1600	1430	83°	104	4°
500	10	32	109	18°	29	68°
	1	320	330	72°	98	20°
200	10	80	125	40°	63	50°
	1	800	830	79°	104	7°
100	10	160	192	58°	83	32°
	1	1600	1430	86°	100	4°
50	10	320	330	68°	95	18°

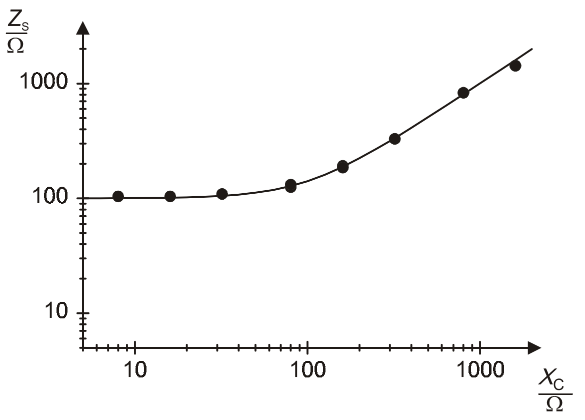


Fig. 5 Total impedance Z_S of the series connection of a capacitor and a 100 Ω resistor as a function of the capacitive reactance X_C

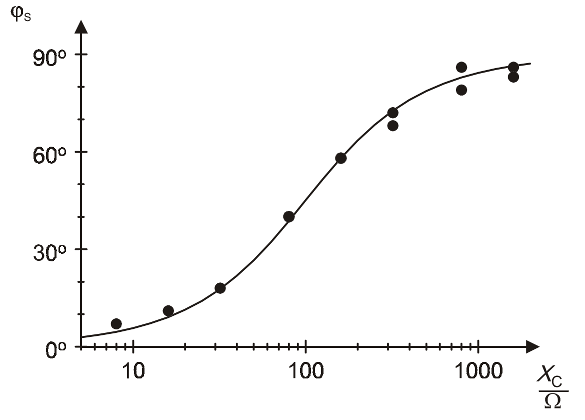


Fig. 6 Phase shift φ_S between the current and the voltage for the series connection of a capacitor and a 100 Ω resistor as a function of the capacitive reactance X_C

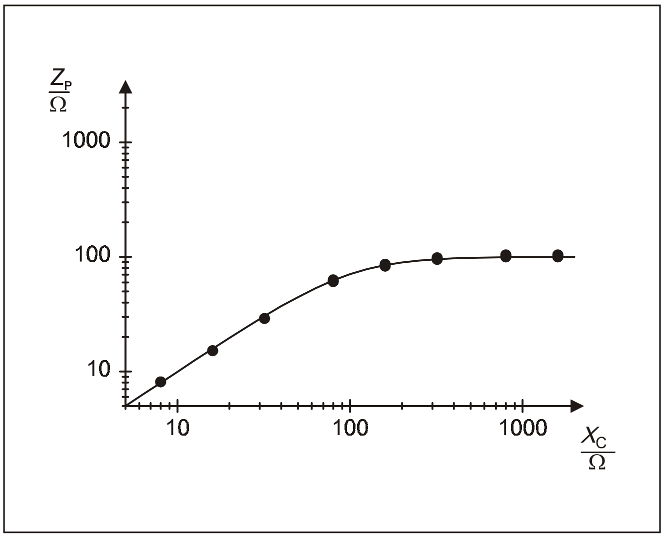


Fig. 7 Total impedance Z_S of the parallel connection of a capacitor and a 100 Ω resistor as a function of the capacitive reactance X_C

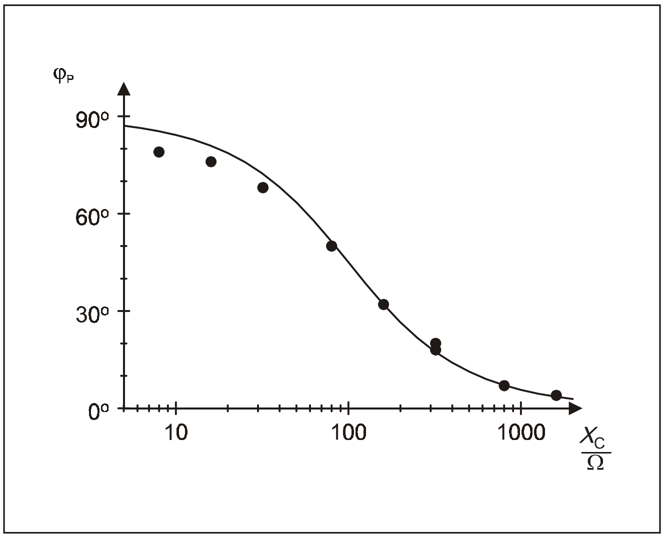


Fig. 8 Phase shift φ_S between the current and the voltage for the parallel connection of a capacitor and a 100 Ω resistor as a function of the capacitive reactance X_C

Supplementary information

The mathematical description of the series and parallel connection of an ohmic resistance and a capacitive reactance becomes more elegant if complex quantities are considered:

When a voltage

$$U = U_0 \cdot e^{i \cdot \omega \cdot t}$$

is applied to a capacitor, the capacitive reactance is

$$X_C = \frac{1}{i \cdot \omega \cdot C}.$$

The impedance Z_S of a series connection of an ohmic resistance R and a capacitive reactance then is

$$Z_S = R + \frac{1}{i \cdot \omega \cdot C}.$$

In the case of a parallel connection, the following relation holds for the total impedance Z_P:

$$\frac{1}{Z_P} = \frac{1}{R} + i \cdot \omega \cdot C.$$