Charging and discharging a capacitor when switching DC on and off

Objects of the experiments

- Investigating the behaviour of the voltage at a capacitor when a DC voltage is switched on and off.
- Determining the half-time $T_{1/2} = R \cdot C \cdot \ln 2$

Principles

In a DC circuit, a capacitor represents an infinite resistance. Only during circuit closing and opening does a current flow. When the circuit is closed, this current causes the capacitor to be charged until the applied voltage is reached. Correspondingly the capacitor is discharged via a resistor when the circuit is opened. The behaviour of the voltage at the capacitor can be described by means of an exponential function.

The voltage during circuit opening is given by:

$$U(t) = U_0 \cdot e^{-\frac{t}{\tau}} \qquad \text{(I)}$$

with τ: time constant

The time constant or decay time τ is the time after which the voltage has dropped to the value $\frac{1}{e} \cdot U_0$.

In the half-time $T_{1/2}$ the voltage drops to half the value of U_0. From Eq. (I) it follows that:

$$U(T_{1/2}) = \frac{1}{2} \cdot U_0 = U_0 \cdot e^{-\frac{T_{1/2}}{\tau}}$$

From this

$$T_{1/2} = \tau \cdot \ln 2$$

is obtained. The time constant τ is determined by the resistance R and the capacitance C:

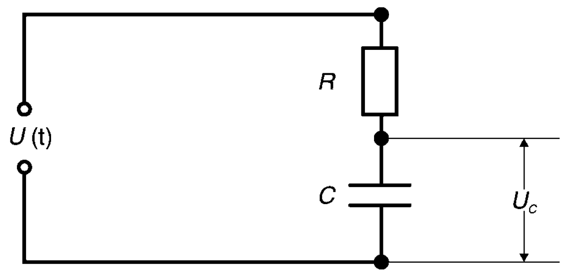
$$\tau = R \cdot C$$

Thus the half-time is:

$$T_{1/2} = R \cdot C \cdot \ln 2 \qquad \text{(II)}$$

Similarly the behaviour of the voltage during circuit closing can be considered:

$$U(t) = U_0 \cdot (1 - e^{-\frac{t}{\tau}})$$



In the experiment, the square-wave voltage $U(t)$ and the voltage U_C at the capacitor C are displayed on the oscilloscope. The half-time $T_{1/2}$ is determined from the time base sweep.

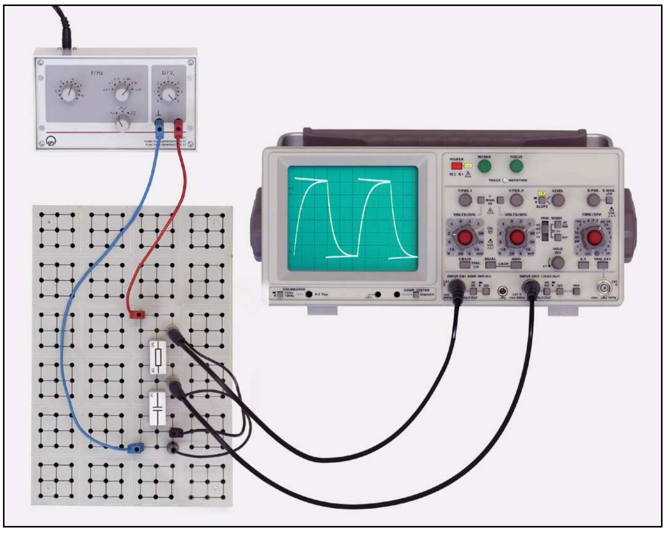
In order to confirm Eq. (II), first the dependence of the half-time $T_{1/2}$ on the capacitance, $T_{\frac{1}{2}} \propto C$, and then on the resistance, $T_{1/2} \propto R$, are investigated.

Apparatus

1 plug-in board A4	576 74
1 resistor 470 Ω, 2 W, STE 2/19	577 40
1 resistor 1 kΩ, 2 W, STE 2/19	577 44
1 resistor 2.2 kΩ, 2 W, STE 2/19	577 48
3 capacitors 1 µF, 100 V, STE 2/19	578 15
1 function generator S 12	522 621
1 two-channel oscilloscope	575 211
2 screened cables BNC/4 mm	575 24
1 pair of cables, 100 cm, blue and red	501 46

Kem 1105

Setup



- Set up the circuit as shown in the picture.
- Measure the square-wave voltage of the function generator with channel I and the voltage drop at the capacitor with channel II.
- Display both curves simultaneously (DUAL). Set the coupling and the trigger to DC. To ensure correct reading of the times *t*, use the calibrated time base sweep (CAL.).

Carrying out the experiment

a) Investigating the discharging and charging processes of a capacitor

- First use the resistor with R = 1 kΩ and one capacitor with C = 1 µF in the setup.
- Select a square-wave voltage with a frequency $f \approx 100$ Hz at the function generator and adjust the voltage $U_S \approx 6$ V with the aid of the oscilloscope so that an even number of fields is used on the screen.
- Measure the times *t* the voltage takes to drop from U = 6 V to U = 3 V and from U = 3 V to U = 1.5 V during discharging.
- Similarly measure the times *t* the voltage takes to rise from 0 V to 3 V and from 3 V to 4.5 V during charging.

b) Dependence of the half-time on the resistance

- Make a capacitance C = 0.5 µF by means of a series connection of two capacitors (1 µF each) and use various resistors one after another.
- For each resistance determine the time $t = T_{1/2}$ in which the voltage U_C at the capacitor has dropped to half the maximum value. If necessary, increase the time base sweep at the oscilloscope for more accurate reading.

c) Dependence of the half-time on the capacitance

- Use the resistor with R = 470 Ω and make various capacitances *C* by means of parallel and series connections of the capacitors.
- For each capacitance determine the time $t = T_{1/2}$ in which the voltage U_C at the capacitor has dropped to half the maximum value. If necessary, increase the time base sweep at the oscilloscope for more accurate reading.

Measuring example

a) Investigating the discharging and charging processes of a capacitor

Table 1: C = 1 µF, R = 1 kΩ (f = 100 Hz)

Voltage change	6 V → 3 V	3 V → 6 V	0 V → 3 V	3 V → 4.5 V
$\frac{t}{\text{ms}}$	0.7	0.7	0.7	0.7

b) Dependence of the half-time on the resistance

Table 2: C = 0.5 µF (f = 100 Hz)

$\frac{R}{\text{k}\Omega}$	0.47	1	1.47	2.2	2.67
$\frac{T_{1/2}}{\text{ms}}$	0.15	0.32	0.47	0.71	0.85

c) Dependence of the half-time on the capacitance

Table 3: R = 0.47 kΩ (f = 100 Hz)

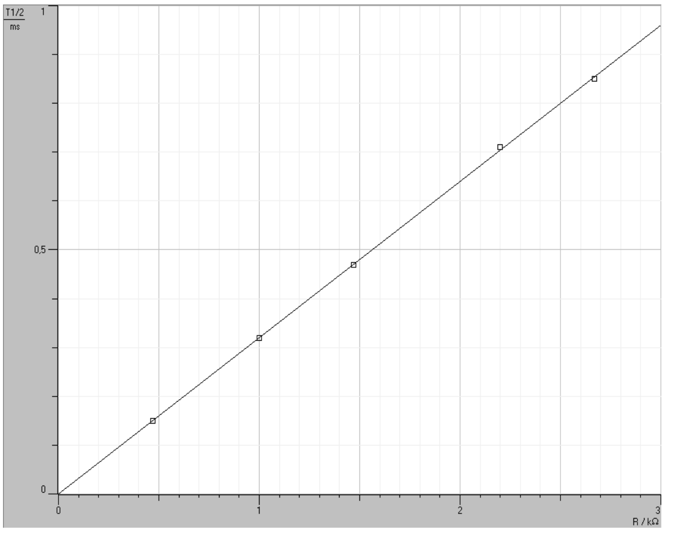
$\frac{C}{\mu\text{F}}$	0.33	0.5	0.67	1	2
$\frac{T_{1/2}}{\text{ms}}$	0.10	0.15	0.21	0.30	0.59

Evaluation and results

a) Investigating the discharging and charging processes of a capacitor

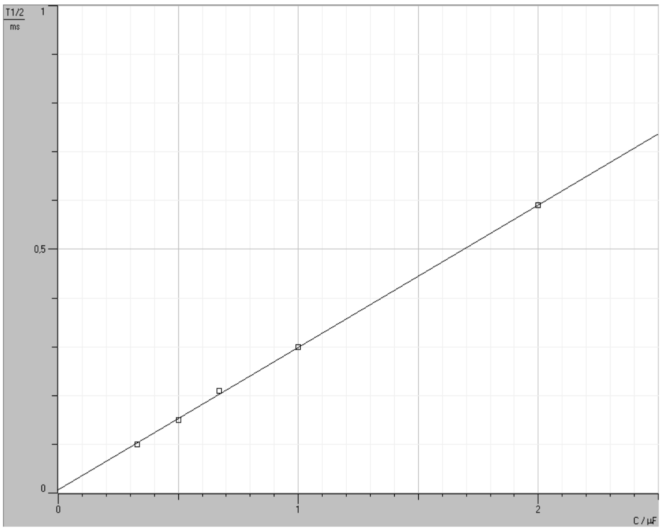
- The time for discharging or charging a capacitor via a resistor is the same if the voltage is just halved or doubled, respectively. Therefore this time is called half-time.

b) Dependence of the half-time on the resistance (C = 1 µF)



The following proportionality is confirmed: $T_{1/2} \propto R$ (III)

c) Dependence of the half-time on the capacitance (R = 0.47 kΩ)



The following proportionality is confirmed: $T_{½} \propto C$ (IV)

d) Determining the constant of proportionality

From (III) and (IV) one gets: $T_{½} \propto R \cdot C$

and thus:

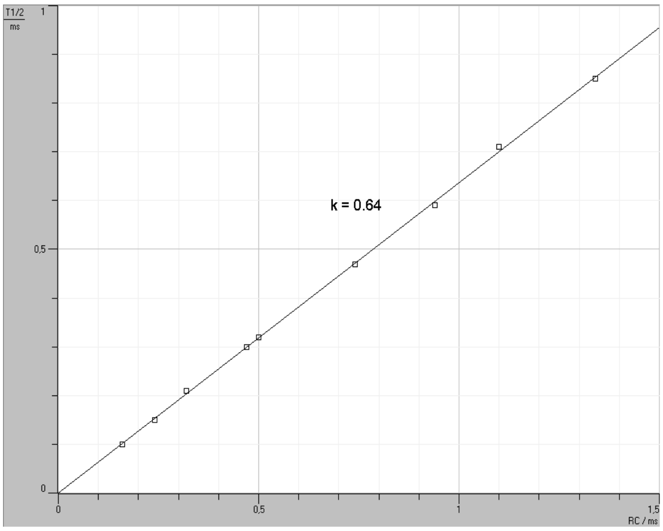
$$T_{½} = k \cdot R \cdot C$$

The values from Tables 2 and 3 give:

$\frac{R \cdot C}{\text{k}\Omega \cdot \mu\text{F}}$	$\frac{T_{½}}{\text{ms}}$
0.16	0.10
0.24	0.15
0.32	0.21
0.47	0.30
0.50	0.32
0.74	0.47
0.94	0.59
1.10	0.71
1.34	0.85

Consideration of units:

$$[R \cdot C] = \text{k}\Omega \cdot \mu\text{F} = 10^3 \frac{\text{V}}{\text{A}} \cdot 10^{-6} \frac{\text{As}}{\text{V}} = 10^{-3}\text{s} = \text{ms}$$



Form the diagram one obtains: $k = 0.64$

Theoretical value: $k = \ln 2 = 0.69$

Apart from the tolerances of the resistors and capacitors, the main reason for the deviation is the inaccurate reading from the oscilloscope.