

## Determining the capacitance of a plate capacitor – Measuring the charge with the electrometer amplifier

### Experiment Objectives

- Measuring the charge  $Q$  on a plate capacitor as a function of the applied voltage  $U$ .
- Determining the capacitance  $C$  as a function of the area  $A$  of the plates.
- Determining the capacitance  $C$  with different dielectrics between the plates.
- Determining the capacitance  $C$  as function of the distance  $d$  between the plates.

### Principles

The simplest design of a capacitor is that of a plate capacitor. Its capacitance

$$C = \frac{Q}{U} \quad (I)$$

$Q$ : charge on the capacitor,  $U$ : applied voltage

depends on the area  $A$  of the plates, the distance  $d$  between the plates and the – non-conductive – material between the plates.

The capacitance of a plate capacitor is

$$C = \epsilon_r \cdot \epsilon_0 \cdot \frac{A}{d} \quad (II),$$

$$\epsilon_0 = 8.85 \cdot 10^{-12} \frac{\text{As}}{\text{Vm}} : \text{permittivity of free space,}$$

as long as the distance between the plates is much smaller than the dimensions of the plates and the field  $E$  between the plates can be considered to be homogeneous. The permittivity  $\epsilon_r$  describes the change of the capacitance relatively to the vacuum value caused by the introduction of the material.

The relation (II) is studied in the experiment by means of a demountable capacitor with variable geometry. Capacitor plates with areas  $A = 40 \text{ cm}^2$  and  $A = 80 \text{ cm}^2$  are available. The distance  $d$  between the plates can be increased with spacers in steps of 1 mm. First the charge  $Q$  on the capacitor is measured as a function of the voltage  $U$ . The capacitance  $C$  is then determined as the slope of the straight line through the origin and through the data points. In order to confirm the proportionality

$$C \propto A \quad (III),$$

which is derived from Eq. (II), the measurement is carried out at a fixed distance  $d$  with different areas  $A$  of the plates.

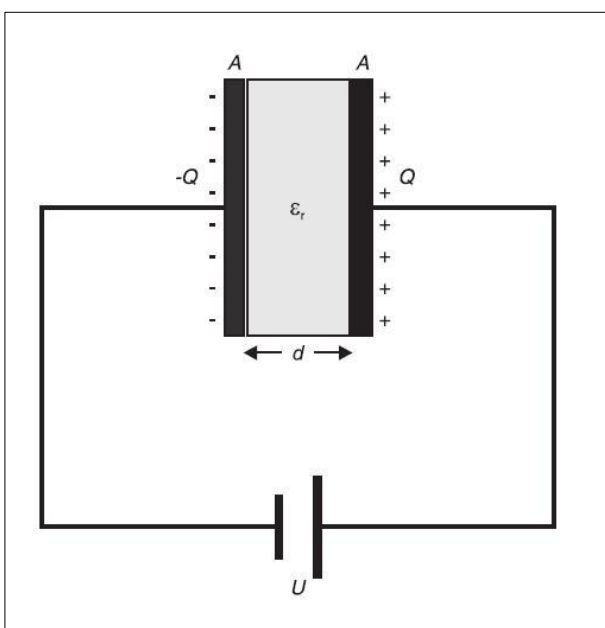
In addition, the permittivities  $\epsilon_r$  of different dielectrics are determined by placing the dielectrics between the capacitor plates. Variation of the distance  $d$  between the plates at a constant area  $A$  allows to confirm the proportionality

$$C \propto \frac{1}{d} \quad (IV).$$

Charges are measured with an electrometer amplifier operated as a coulombmeter. Any voltmeter may be used to display the output voltage  $U_A$ . From the reference capacitance  $C_A$

$$Q = C_A \cdot U_A \quad (V)$$

is obtained. For example, at  $C_A = 10 \text{ nF}$ ,  $U_A = 1 \text{ V}$  corresponds to the charge  $Q = 10 \text{ nAs}$ . If other capacitances are used, other measuring ranges are accessible.



**Apparatus**

1	Demountable capacitor .....	544 23
1	Power supply 450 V .....	522 27
1	Electrometer amplifier .....	532 14
1	Capacitor 10 nF, STE 2/19 .....	578 10
1	Capacitor 0.1 μF, STE 2/19 .....	578 31
1	Connecting rod .....	532 16
2	Multimeter LD analog 20 .....	531 120
1	Two-way switch .....	504 48
4	Connecting lead 19 A, 50 cm, red/blue, pair ..	501 45
2	Connecting lead 19 A, 100 cm, red/blue, pair	501 46

**Setup**

The experimental setup is illustrated in Fig. 1.

- Mount the pair of small plates ( $A = 400 \text{ cm}^2$ ), and set the distance  $d$  between the plates to 4 mm with the spacers.
- Connect the negative pole of the power supply 450 V to the right plate and to the earth of the electrometer amplifier. Connect the connection rod to the earth as well.
- Connect the positive pole of the power supply 450 V to socket B of the two-way switch.
- Connect socket A of the two-way switch to the left plate and socket C to the input of the electrometer amplifier.
- Plug the reference capacitor  $C_A = 10 \text{ nF}$  in at the electrometer amplifier, and connect the voltmeter to the output.
- Connect the other voltmeter to the power supply 450 V for measuring the voltage  $U$ .

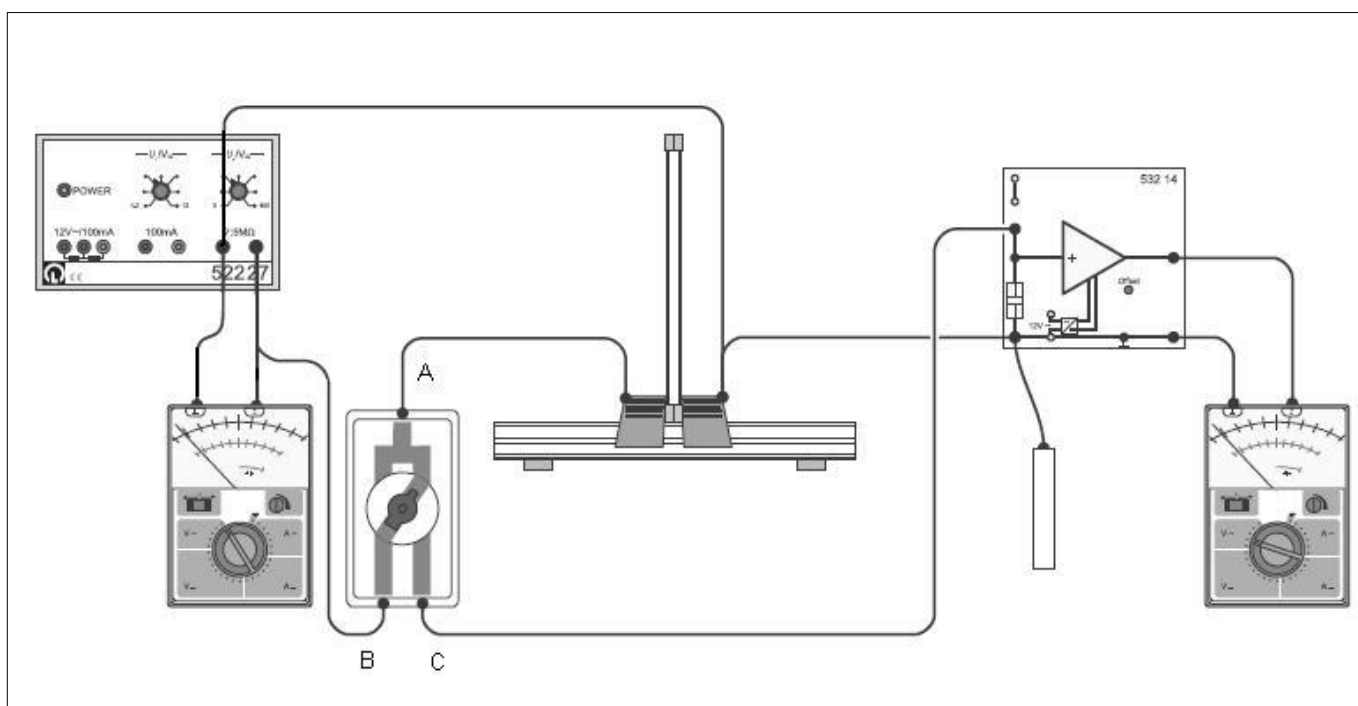


Fig. 1 Experimental setup for the determination of the capacitance of a plate capacitor

### Carrying out the Experiment

#### a) Measuring the charge as a function of the voltage for different areas of the plates:

- Establish the connection AC with the two-way switch, discharge the plate capacitor with the connection rod, and check the zero of the charge measurement.
- Hold the connection rod in your hand, change to the connection AB with the two-way switch, and set the output voltage  $U$  to 50 V.
- Change back to the connection AC, measure the charge  $Q$  with the electrometer amplifier, and take the charge down.
- Repeat the measurement with other voltages.
- Replace the pair of small plates with the pair of large plates ( $A = 800 \text{ cm}^2$ ,  $d = 4 \text{ mm}$ ).
- Establish the connection AC, and discharge the capacitor with the connection rod.
- Hold the connection rod in your hand, and record the second series of measurements.

#### b) Measuring the charge as a function of the voltage for different dielectrics:

- Place the polystyrene plate between the pair of large plates, and see to it that the surfaces of the capacitor plates are in contact with the polystyrene plate.
- Establish the connection AC with the two-way switch, and discharge the capacitor with the connection rod.
- Hold the connection rod in your hand, and measure the charge  $Q$  as a function of the voltage  $U$ .
- Plug the capacitor  $C_A = 100 \text{ nF}$  ( $0.1 \text{ }\mu\text{F}$ ) in at the electrometer amplifier.
- Replace the polystyrene plate with the glass plate, establish the connection AC, and discharge the capacitor with the connection rod.
- Hold the connection rod in your hand, and record the next series of measurements.

#### c) Determining the capacitance as a function of the distance between the plates:

- Set the voltage  $U$  to 300 V.
- Remove the glass plate and set the distance  $d$  between the plates to 6 mm with the spacers.
- Establish the connection AC with the two-way switch, and discharge the capacitor with the connection rod.
- Hold the connection rod in your hand, change to the connection AB to charge the capacitor, then change back to the connection AC for the charge measurement.
- Read the charge  $Q$  and record it.
- Subsequently reduce the distance between the plates to 4, 3, 2 and 1 mm, recharge the capacitor, and measure the charge.

### Measurement Example

#### a) Measuring the charge as a function of the voltage for different areas of the plates:

Table 1: the charge  $Q$  recorded as a function of the applied voltage  $U$  with different areas  $A$  of the plates:

$\frac{U}{V}$	$\frac{Q}{\text{nAs}}$ ( $A = 400 \text{ cm}^2$ )	$\frac{Q}{\text{nAs}}$ ( $A = 800 \text{ cm}^2$ )
50	5	10.5
100	10.5	20
150	16	30
200	21.5	41
250	26.5	51
300	33	59

#### b) Measuring the charge as a function of the voltage for different dielectrics:

Table 2: The charge  $Q$  recorded as a function of the applied voltage  $U$  for different dielectrics ( $A = 800 \text{ cm}^2$ )

$\frac{U}{V}$	$\frac{Q}{\text{nAs}}$	$\frac{Q}{\text{nAs}}$
	polystyrene	glass
50	15	40
100	32	84
150	46	126
200	62	164
250	78	208
300	95	249

#### c) Determining the capacitance as a function of the distance between the plates:

Table 3: The charge  $Q$  (at  $U = 300 \text{ V}$ ) and the capacitance  $C$  as functions of the distance  $d$  between the plates

$\frac{d}{\text{mm}}$	$\frac{Q}{\text{nAs}}$	$\frac{C}{\text{pF}}$
1	100	333
2	52	173
3	35	116
4	26	86
6	16	53

**Evaluation and results**

**a) Measuring the charge as a function of the voltage for different areas of the plates:**

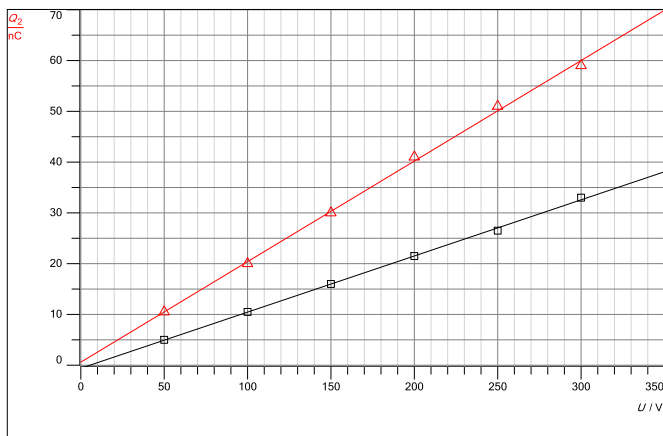


Fig. 2 The charge  $Q$  on the capacitor as a function of the applied voltage  $U$ .  
Area  $A$  of the plates:  $400 \text{ cm}^2$  (square),  $800 \text{ cm}^2$  (triangle)

Fig. 2 is a plot of the measuring values of Table 1. Within the accuracy of measurement the measuring values lie on the straight line through the origin drawn in the graph. According to Eq. (I), the slope of this straight line (see Table 4) is the capacitance  $C$ .

The ratio of the capacitances in air (see Table 4) agrees to a good approximation with the ratio of the areas. Thus the proportionality  $C \propto A$  is confirmed.

**b) Measuring the charge as a function of the voltage for different dielectrics:**

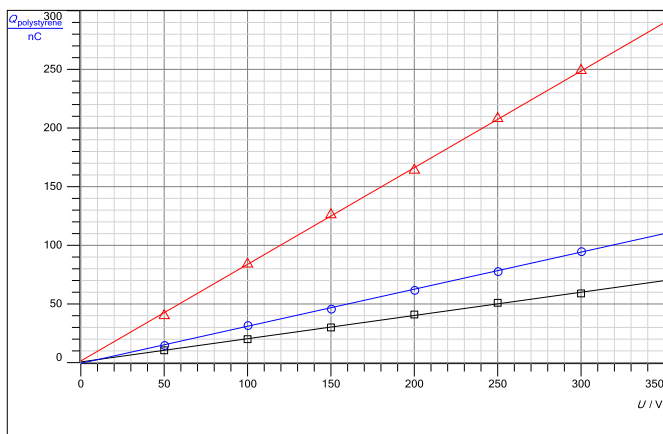


Fig. 3 The charge  $Q$  on the capacitor as a function of the applied voltage  $U$   
dielectric: air (square), polystyrene (circle), glass (triangle)

Fig. 3 is a plot of the measuring values of Table 2. For comparison the measuring values corresponding to the larger area of the plates from Table 2 are also plotted. Within the accuracy of measurement the measuring values lie on the straight line through the origin drawn in the graph. The slope of this straight line (see Table 4) is the capacitance  $C$  according to Eq. (I).

The capacitance of the capacitor increases if there is a dielectric between the plates. Now the permittivities of polystyrene and glass can be determined from the measuring values.

For polystyrene  $\epsilon_r = 1.6$  and for glass  $\epsilon_r = 4.2$  is obtained.

Table 4: Evaluation of Figs. 2 and 3

dielectric	$\frac{A}{\text{cm}^2}$	$\frac{C}{\text{pF}}$
air	400	111
air	800	198
polystyrene	800	316
glass	800	824

**c) Determining the capacitance as a function of the distance between the plates:**

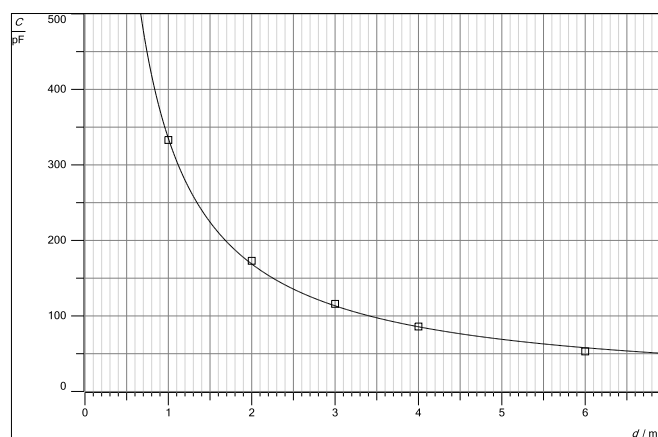


Fig. 4 The capacitance  $C$  of the plate capacitor as a function of the distance  $d$  between the plates.

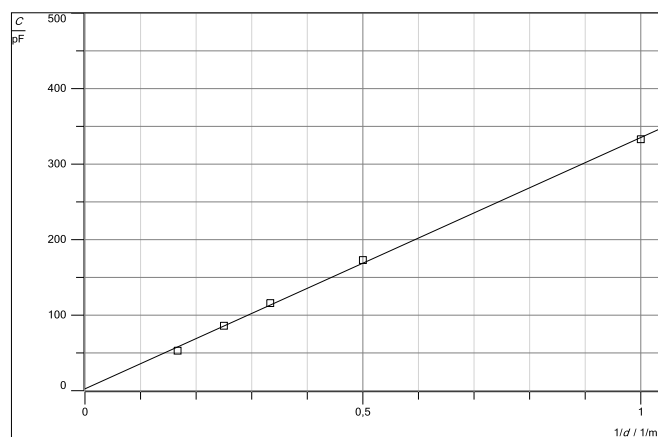


Fig. 5 The capacitance  $C$  of the plate capacitor as a function of  $1/d$

Figs. 4 and 5 are plots of the measuring values of Table 4. It can be seen that the capacitance of the plate capacitor does not decrease linearly with decreasing distance  $d$  between the plates. In Fig. 5, the measuring values lie on the straight line through the origin. Thus the proportionality

$$C \propto \frac{1}{d} \text{ is confirmed.}$$