Electricity

Electrostatics

Field lines and equipotential lines

Physics Leaflets

P3.1.3.4

Measuring the potential around a charged sphere

Objects of the experiments

- To measure the potential at equal distances from the sphere: equipotential surfaces
- To measure the potential as a function of the distance from the surface of the sphere and determination of the electrical field strength

Principles

In the two dimensional section through an electric field the points of equal potential form a line. The course of such equipotential lines is, just like the course of the field lines, determined by the spatial arrangement of the field-generating charges. The equipotential lines are always perpendicular to the electric field lines.

In the experiment the potential around a charged sphere is investigated. The equipotential lines lie, if viewed three-dimensionally, on spherical surfaces, if viewed two-dimensionally in circles around the sphere (see fig. 1). The following applies:

$$U = \frac{Q}{4\pi\varepsilon_0 \cdot r} \qquad \text{(I)}$$

The capacitance of a charged sphere with a radius *a* is given by the ratio of voltage *U* and charge *Q* at the surface of a sphere:

$$C = \frac{Q}{U} = 4\pi\varepsilon_0 \cdot a \qquad \text{(II)}$$

The size of the electric field *E* is given by the change of the potential *U* in space. The electric field strength *E* therefore depends on the distance *d* from the sphere surface or on the distance *r* from the centre of the sphere.

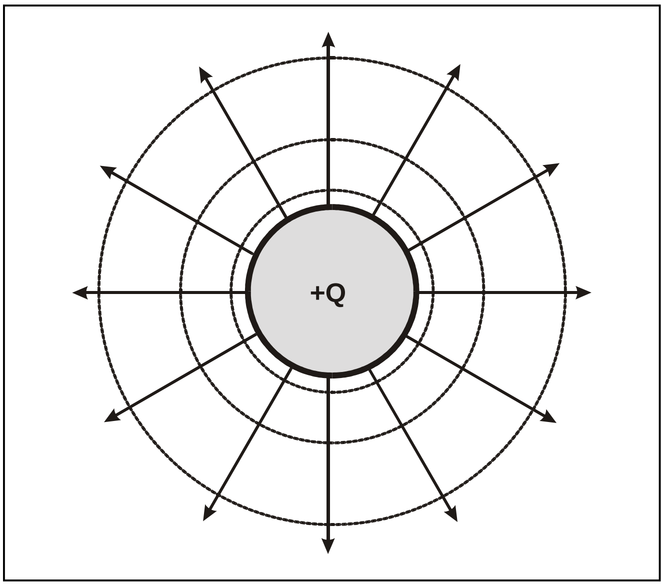

Fig. 1: Electric field and equipotential lines around a charged sphere

Fig. 2: Experimental setup

The electric field lines run radially-symmetrically from the sphere (see fig. 1). The following applies:

$$E = \frac{Q}{4\pi\varepsilon_0 \cdot r^2} \qquad \text{(III)}$$

For measuring the potential around the sphere a flame probe is used. In the metal tube of the flame probe a flammable gas flows to the tip and burns there with a small flame. Because of the flame at the tip, an ionisation current will flow until the ambient potential is reached. The resulting voltage is transmitted via the connection cable to the voltage measuring panel on the electric field meter S and there it is measured.

In this experiment the potential around the sphere is initially determined by moving the flame probe step by step at the desired distance *d* around the sphere and at each step the voltage *U* is measured.

Then the flame probe is moved stepwise radially away from the surface of the sphere and again the voltage *U* is measured for each step. The voltage values are plotted as a function of their distance from the centre of the sphere *r*. In addition the function *U* ~ *1/r* is checked by plotting the voltage values against *1/r* and fitting a straight line to the values. The

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gradient of this straight line is used for determining the charge Q. By means of equation (II) *U* and *Q* are used for calculating the capacitance *C* of the charged sphere and it is compared to the theoretical value $C = 4\pi\varepsilon_0 \cdot a$.

For determining the electric field *E*, the change of the potential as a function of the change of the distance is determined and plotted The relationship U ~ $1/r^2$ is checked by plotting the values against $1/r^2$ and fitting a straight line to the values.

Setup

The experimental setup is shown in fig. 2. For setting up the following steps are required:

- Fit the sphere with the connection cable into a base.
- The electric field meter and the flame probe are also fitted into a base.
- **Earth the left-hand positive pole of the 10 kV high voltage power supply and connect it to the earthing socket on the back of the electric field meter.**
- Connect the left-hand negative pole of the 10 kV high voltage meter to the sphere with the connection cable.
- Connect the electric field meter to the universal measuring instrument P and select “Voltage” as the measuring unit.
- Place the voltage measuring plate onto the electric field meter and connect the flame probe to the voltage measuring plate.
- Connect the cartridge to the flame probe, check the firmness of the tube connections.
- Hold a burning lighter or match to the tip of the flame probe and slowly open the gas supply until a small flame, approx. 10 mm high burns at the tip.
- During the measurement hold the flame probe only by its insulating stand rod, because any contact with metal parts will prevent the potential equalisation.

Note:

- Do not put the flame probe into excessively strong fields; as this will cause the flame to become sooty.
- Placing the flame into a setup reduces the dielectric strength of the air and may therefore lead to electric flashovers.
- **Set up metal objects (e.g. high voltage power supply, universal measuring instrument P) as far away from the sphere as possible in order to disturb the potential and electric field around the charged sphere as little as possible.**

Warning:

It is absolutely necessary to provide correct earthing of the electric field meter S. Because typically the measurement is made using a high voltage, the electric field meter S must never be operated without the 4 mm socket on the back being connected to ground. When it is connected correctly the current flows back to the power supply should the voltage flash over and not to the meter.

Should the earthing not be correct, peripheral equipment (e.g. the meter or Sensor-CASSY) connected to the electric field meter S could become damaged!

Apparatus

1 sphere with connection cable	543 08
1 electric field meter S	524 080
1 set of accessories for the electric field meter S	540 540
1 universal measuring instrument P	531 835
1 high voltage power supply 10 kV	521 70
3 saddle bases	300 11
1 wooden ruler, L = 1m / 39 inch	311 03
or	
2 clamp riders with clamp 45/35	460 312
1 optical bench, S1 profile, 50 cm	460 317
1 saddle base	300 11
1 safety connection lead, 10 cm, yellow/green	500 600
1 safety connection lead, 100 cm, red	500 641
1 safety connection lead, 100 cm, blue	500 642
1 cartridge	666 715
1 valve for gas cartridge	666 716
1 PVC tubing, 7 x 1.5 mm, 1 m	667 193

Note:

For carrying out this experiment, as an alternative to the universal measuring instrument P the following can be used:

1 mobile CASSY (524 009)

or

1 Sensor-CASSY (524 010USB) + CASSY Lab (524 200)) / CASSY-Display (524 020)

or

1 Pocket CASSY (524 009) + CASSY Lab (524 200)

Carrying out the experiment

a) Measurements at a constant distance from the sphere

- When the high voltage is switched off, adjust the desired distance *d* between the flame probe and the sphere. Hold the flame probe perpendicular to the connection line between the flame and the sphere so that the field around the sphere is disturbed as little as possible by the flame probe.
- Increase the high voltage at the sphere to 3.0 kV.
- Note down the measured voltage *U* and the position.
- Change the position of the flame probe relative to the sphere by constant intervals *d* and repeat the measurement several times.

b) Measurement as a function of the distance from the sphere surface and determination of the electric field strength

- Set the saddle base with the flame probe on the wooden ruler in such a way that the flame probe can be moved radially away from the sphere.
- Determine the distance of the flame probe from the sphere.
- Increase the high voltage at the sphere to 3.0 kV.
- Move the flame probe on the wooden ruler step by step perpendicular to the capacitor plates and note for each step the measured voltage *U* and the position.

Measuring example and evaluation

a) Measurements at a constant distance from the sphere

In the example, the potential was measured at various positions around the charged sphere. The charge was connected to approx. -3.0 kV and the distance to the sphere for each of the measurements was 2.0 cm. For the potential, values between –2.44 kV and –2.47 kV were found. This means that the measuring points are approximately on an equipotential surface, which in the case of a charge sphere has the shape of a sphere.

b) Measurement as a function of the distance to the sphere surface and determination of the electric field strength

In table 1 the results of an example of a measurement are shown. The sphere was charged to –3.0 kV. The location x = 0 mm corresponded to the surface of the sphere. In order to maintain the distance r of the flame probe from the centre of the sphere, the sphere radius a = 5 cm is added to the distance x from the surface of the sphere.

In figure 3 the voltage values are plotted as a function of their distance from the centre of the sphere r. The value measured continuously reduces with increasing distance from the charged sphere, i.e. the potential decreases.

For checking the dependency $U \sim 1/r$ the values $1/r$ are also determined and the voltage is plotted as a function of the latter (figure 4). The fit of a straight line shows a good match with the measured data and confirms therefore the relationship $U \sim 1/r$. From the gradient A = 180 Vm of the straight line using equation (I) the charge on the sphere is also able to be determined.

$$Q = 4\pi\varepsilon_0 \cdot A = -2{,}00 \cdot 10^{-8}\,\text{C} = -1{,}25 \cdot 10^{11} e$$

with the elementary charge $e = 1{,}6022 \cdot 10^{-19}\,\text{C}$.

r / cm	6	7	8	9	10	11
U / kV	-2.76	-2.46	-2.15	-1.89	-1.69	-1.52
r / cm	12	13	14	15	16	17
U / kV	-1.38	-1.27	-1.17	-1.08	-1.00	-0.92
r / cm	18	19	20	22.5	25	27.5
U / kV	-0.86	-0.80	-0.75	-0.65	-0.58	-0.51
r / cm	30	32.5	35	37.5	40	45
U / kV	-0.48	-0.39	-0.36	-0.33	-0.30	-0.28

Tab. 1: Measuring result of the potential U

r_m / cm	6.5	7.5	8.5	9.5	10.5	11.5
E / kV/m	-30	-31	-26	-23	-17	-14
r_m / cm	12.5	13.5	14.5	15.5	16.5	17.5
E / kV/m	-11	-10	-9	-8	-8	-6
r_m / cm	18.5	19.5	21.25	23.75	26.25	28.75
E / kV/m	-6	-5	-4	-2.8	-2.8	-1,2
r_m / cm	31.25	33.75	36.25	38.75	42.5	
E / kV/m	-3.6	-1.2	-1.2	-1.2	-0.4	

Tab. 2: Data relating to the electric field strength E

From the charge Q on the sphere and the applied voltage U the capacitance of the sphere can also be determined (equation II). With $Q = -2{,}00 \cdot 10^{-8}\,\text{C}$ and $U = -3{,}0\,\text{kV}$ one obtains $C = 6{,}7\,\text{pF}$. For a sphere with a radius of a = 5 cm this theoretically gives $C = 4\pi\varepsilon_0 \cdot a$ = 5.6 pF. The deviation from the measured result arises mainly from the assumption that the potential only reaches zero at infinity. This is in the experiment only true in approximation, because e.g. measuring instruments cannot be placed at an infinite distance from the sphere and close to the experimental setup there are further surfaces such as the table top. These influence the potential and therefore lead to deviations in the measurements.

For determining the electric field strength E the change of the potential as a function of the change of the distance is determined: $E = \frac{\Delta U}{\Delta r}$

The values are plotted as a function of the mean distance r_m, i.e. as the mean of the two distances from which the Δr is determined. The data are shown in table 2 and figure 5.

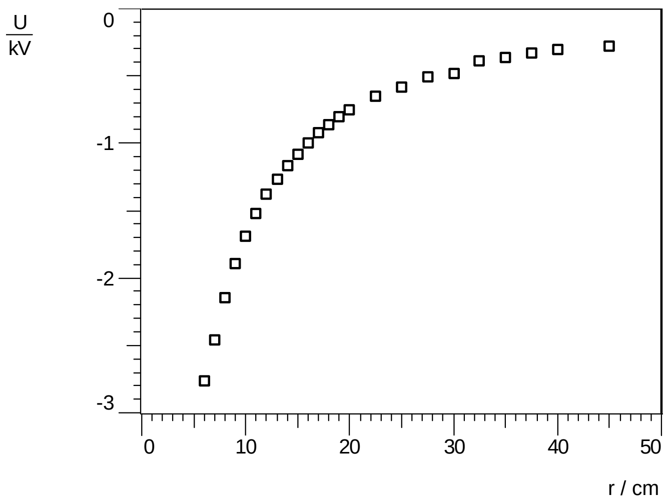

Fig. 3: Voltage U as a function of the distance r from the centre of the sphere

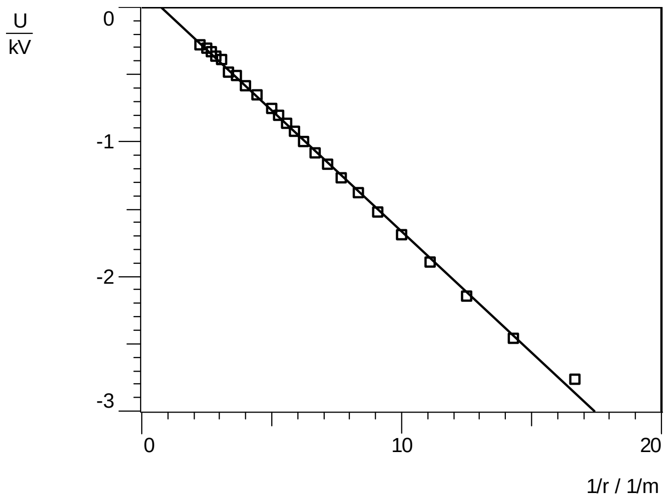

Fig. 4: Voltage U as plotted against $1/r$ (r: distance from the centre of the sphere)

For checking the dependency U ~ $1/r^2$ the values $1/r_m^2$ are also determined and the voltage is plotted as a function of the latter (figure 6); the fit of a straight line shows a good match with the measured data and confirms therefore the relationship U ~ $1/r^2$.

The gradient A = 184 Vm also corresponds to $\frac{Q}{4\pi\varepsilon_0}$.

Within the limitations of the measuring precision the two values one obtains from the evaluation of the potential function and the shape of the field lines agree. The relative error is larger for the field values than for the potential values because for this values of a similar order of magnitude have to be subtracted from each other. This effect is clearly apparent in the strong scatter of the values for the electric field strength E at large distances from the sphere centre.

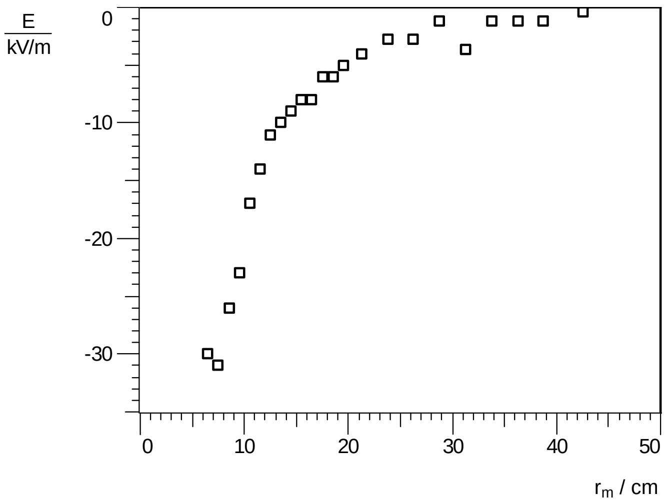

Fig. 5: Electric field strength E as a function of the distance r_m from the centre of the sphere

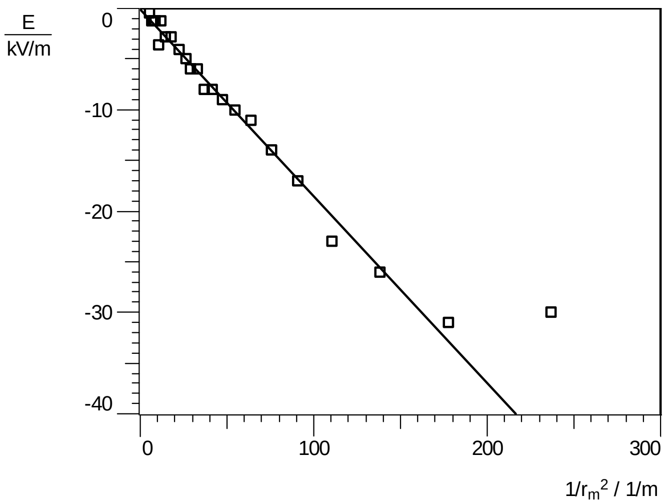

Fig. 6: Electric field strength E plotted against $1/r_m^2$