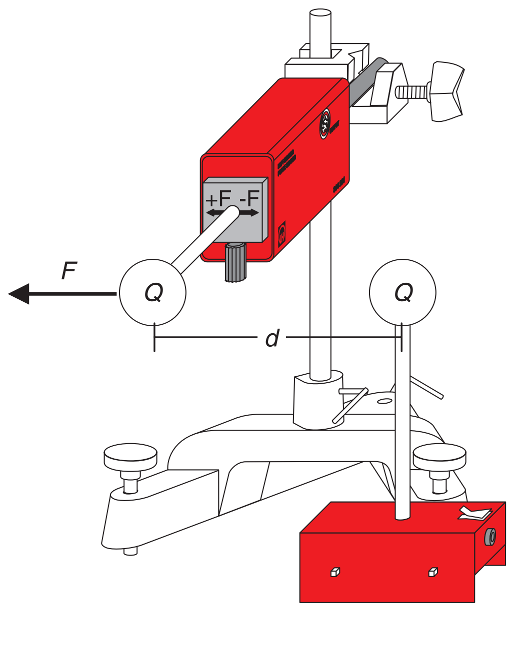
Confirming Coulomb's law

Measuring with the force sensor and newton meter

Objects of the experiments

- Measuring the force F between two charged balls as a function of the distance d between the balls.
- Measuring the force F between two charged balls as a function of their charges Q_1 und Q_2.
- Estimating the permittivity of free space ε_0.



Principles

According to Coulomb's law, the force between two pointlike charges Q_1 and Q_2 at a distance d is

$$F = \frac{1}{4\pi \cdot \varepsilon_0} \cdot \frac{Q_1 \cdot Q_2}{d^2} \qquad \text{(I)}$$

with $\varepsilon_0 = 8.85 \cdot 10^{-12} \frac{\text{As}}{\text{Vm}}$: permittivity of free space

The force F is positive, that is repulsive, if both charges have the same sign. If the signs of the charges are different, the force is negative, that is attractive.

The force between two charged spheres is approximately the same if the distance d between the centres is considerably larger than the radii r of the spheres so that the uniform charge distribution on the spheres remains undistorted. At smaller distances d, measuring results are changed by an "image force" caused by mutual electrostatic induction.

The force between two charged balls will be measured in the experiment by means of a force sensor. You will study the proportionalities

$$F \propto \frac{1}{d^2},\ F \propto Q_1 \text{ and } F \propto Q_2 \qquad \text{(II).}$$

The device for measuring the force contains two parallel flection elements and four strain gauges connected in bridge circuit. The electric resistance of the strain gauges changes under mechanical stress. This change in resistance is proportional to the acting force, which is directly displayed by a newtonmeter.

An electrometer operated as a coulombmeter enables the charges on the balls to be measured almost without current. Any voltmeter may be used to display the output voltage U_A. From the reference capacitance C

$$Q = C \cdot U_A \qquad \text{(III)}$$

is obtained. For example, at C = 10 nF, U_A = 1 V corresponds to the charge Q = 10 nAs. If other capacitances are used, other measuring ranges are accessible.

0909-Wie

Apparatus

1 set of bodies for electric charge		314 263
1 trolley 1.85 g		337 00
1 precision metal rail, 0.5 m		460 82
1 force sensor		314 261
1 newtonmeter		314 251
1 multicore cable, 6-pole, 1.5 m		501 16
1 high voltage power supply 25kV		521 721
1 high voltage cable, 1 m		501 05
1 insulated stand rod, 25 cm		590 13
1 saddle base		300 11
1 electrometer amplifier		532 14
1 plug-in unit 230 V/12 V AC/20 W		562 791
1 STE capacitor 1 nF, 630 V		578 25
1 STE capacitor 10 nF, 100 V		578 10
1 voltmeter, up to U = ±8 V	f.e.	531 100
1 Faraday's cup		546 12
1 clamping plug		590 011
1 connection rod		532 16
1 stand base, V-shape		300 02
1 stand rod, 25 cm		300 41
1 Leybold multiclamp		301 01
connection leads		

Safety notes

The high voltage power supply 25 kV fulfills the safety requirements for electrical equipment for measurement, control and laboratory. It supplies a non-hazardous contact voltage. Observe the following safety measures.

- Observe the instructions of the high voltage power supply.
- Always make certain that the high voltage power supply is switched off before altering the connections in the experimental setup.
- Set up the experiment so that neither non-insulated parts nor cables and plug can be touched inadvertently.
- Always set the output voltage to zero before switching on the high voltage power supply (turn the knob all the way to the left).
- In order to avoid high-voltage arcing, lay the high voltage cable in a way that there are no conductive objects near the cable.

Preliminary remark

Carrying out this experiment requires particular care because "leakage currents" through the insulators may cause charge losses and thus considerable measuring errors. Moreover, undesirable effects of electrostatic induction may influence the results.

The experiment must be carried out in a closed, dry room so as to prevent charge losses due to high humidity.

Cleaning the insulated rods, which hold the balls, with distilled water is recommended because distilled water is the best solvent of conductive salts on the insulators. In addition, the insulated rods should be discharged after every experiment by quickly passing them through a non-blackening flame several times; for example, that of a butane gas burner.

The high voltage power supply and the point of the high voltage cable must be at a sufficient distance from the rest of the experimental setup so as to avoid interference by electrostatic induction.

For the same reason, the experimenter – particularly while measuring charges – must keep the connection rod of the electrometer amplifier in his hand in order to earth himself.

Setup

The experimental setup has two parts. In Fig. 1, the setup for charging the balls and for measuring the force is illustrated. Fig. 2 shows the connection of the electrometer amplifier for the charge measurement.

High voltage supply:

– Connect the high voltage cable to the positive pole of the high voltage power supply and the negative pole to earth.
– Put the free point of the high voltage cable **(a)** through the uppermost hole of the insulated stand rod.

Arrangement of the force sensor and the balls:

– Put the trolley **(b)** onto the precision metal rail, and attach ball 1 by means of the connector.
– Attach the force sensor **(c)** to the stand material so that its "-"-side points at ball 1 (repulsive forces are considered to be positive).
– Attach ball 2 with the insulated rod to the force sensor and lock with the screw.
– Align the two balls at the same height.
– Connect the force sensor to the newtonmeter with the multicore cable.
– Move the trolley so that its left edge coincides with the scale mark 4.0 cm, and set the distance between the balls to 0.2 cm (distance between the centres d = 4.0 cm).

Setup for the charge measurement:

– Supply the electrometer amplifier with voltage from the plug-in unit.
– Attach the Faraday's cup **(d)** with the clamping plug.
– Attach the capacitor 10 nF **(e)**.
– Use a connection lead to connect the connection rod **(f)** to ground and, if possible, the ground to the earth of the high voltage power supply through a long connection lead.
– Connect the voltmeter to the output.

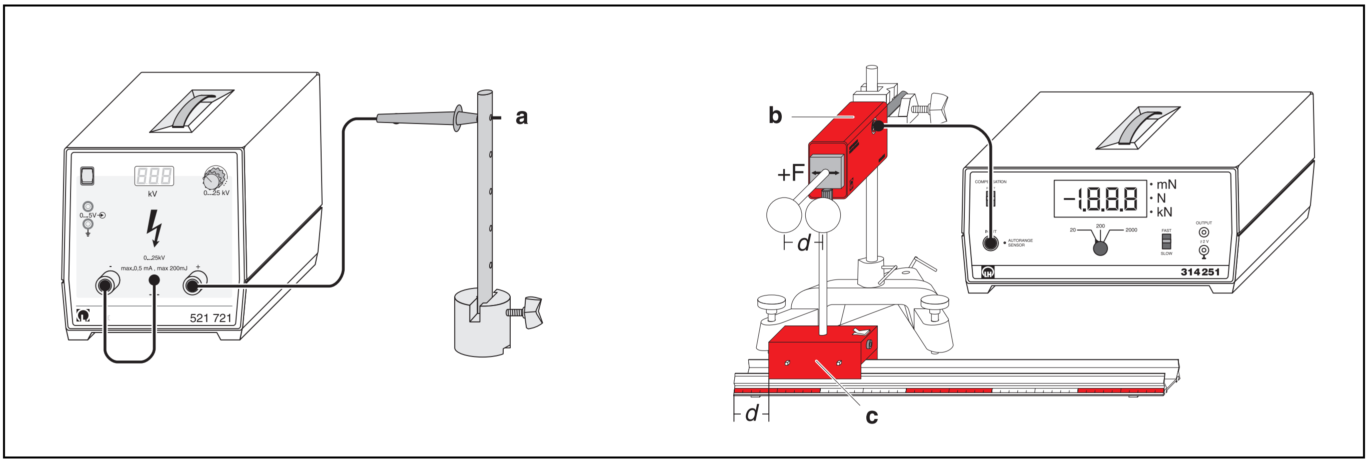


Fig. 1 Setup for measuring the force between two electrically charged balls as a function of their distance.

Carrying out the experiment

Notes:

The measurement is liable to being influenced by interferences from the vicinity because the forces to be measured are very small: Avoid vibrations, draught and variations in temperature.

The newtonmeter must warm up at least 30 min before the experiment is started: switch the newtonmeter on at the mains switch on the back of the instrument to which the force sensor is connected.

a) Measurement at various distances *d* between the balls:

a1) Measurement with equal charges:

- Move ball 1 with the trolley to the maximum distance.
- Switch the high voltage power supply on, and set the output voltage to *U* = 25 kV.
- Touch the two balls successively with the point **(a)** of the high voltage cable.
- Set the high voltage back to zero.
- Make the zero compensation by setting the pushbutton COMPENSATION of the newtonmeter to SET.
- Move ball 1 towards ball 2, measure the force *F* as a function of the distance *d* and take it down.

a2)Measurement with opposite charges:

- Move ball 1 back to maximum distance.
- Make the compensation of the newtonmeter again.
- Charge ball 2 again.
- Set the high voltage back to zero, and change the polarity (high voltage cable at negative pole, positive pole at earth).
- Set the output voltage to *U* = 25 kV and charge ball 1 negatively.
- Move ball 1 towards ball 2, measure the force *F* as a function of the distance *d* and take it down.

b) Measurement with various charges Q_1 and Q_2:

b1) Measurement of the charge on the balls

- Move ball 1 back to maximum distance.
- Set the high voltage back to zero, and change the polarity.
- Charge ball 1 positively with *U* = 25 kV, and set the high voltage back to zero.
- While measuring charges keep the connection rod **(f)** in your hand. Move the ball into the Faraday's cup with the insulated rod (see Fig. 3).

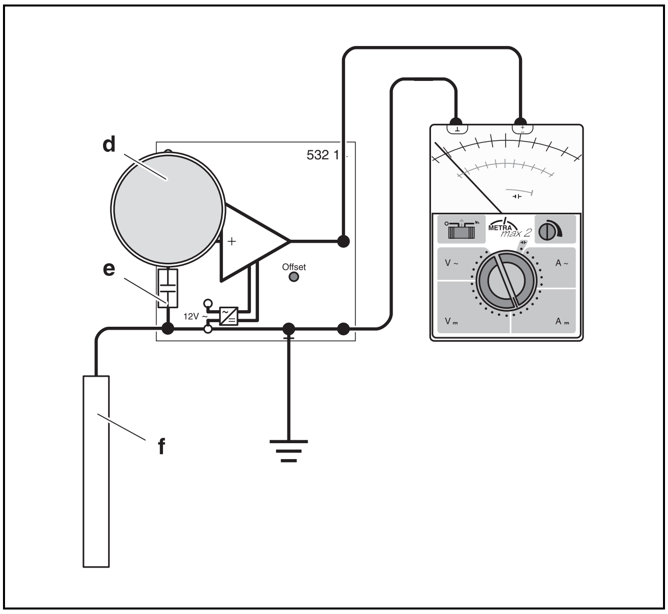
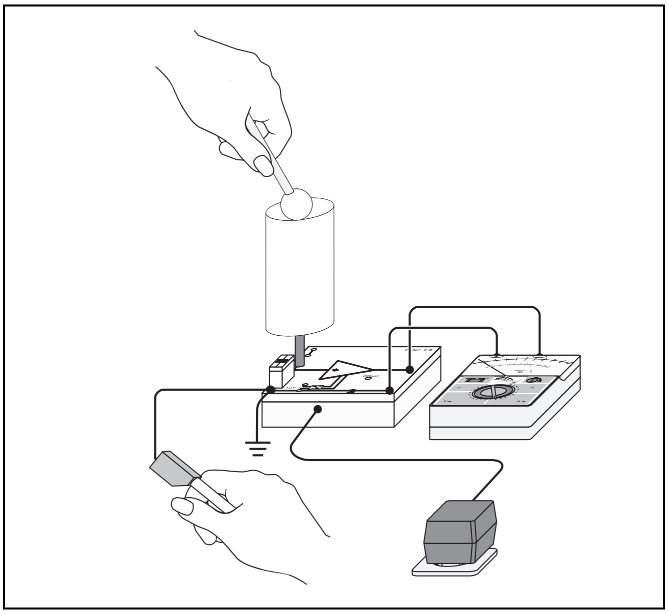


Fig. 2 Connection of the electrometer amplifier for the charge measurement.

Fig. 3 Measurement of the charge on a ball.



– Repeat the measurement at U = 20 kV, U = 15 kV, 10 kV and 5 kV (before each measurement discharge the ball by contact with the connection rod).
– Record the same series of measurements with ball 2.

b2) Measurement of the force F as a function of Q_2 (Q_1 >0, Q_2 >0):

– Mount the two balls again and move ball 1 back to maximum distance.
– Provide for compensation of the newtonmeter again.
– Charge ball 1 with U = 25 kV.
– Charge ball 2 successively with 5 kV, 10 kV, 15 kV, 20 kV and 25 kV with the balls at maximum distance, set the high voltage back to zero each time, choose the distance d = 6 cm and measure the force F.

b3) Measurement of the force F as a function of Q_1 (Q_1 < 0, Q_2 >0):

– Move ball 1 back to maximum distance.
– Provide for compensation of the newtonmeter again.
– Charge ball 2 with U = 25 kV.
– Set the high voltage back to zero and change the polarity.
– Charge ball 1 successively with –5 kV, –10 kV, –15 kV, –20 kV and –25 kV with the balls at maximum distance, set the high voltage back to zero each time, choose the distance d = 6 cm and measure the force F.

Measuring example

a) Measurement at various distances *d* between the balls:

Table 1: The Coulomb force F between two balls as a function of the distance d

$\frac{d}{\text{cm}}$	$\frac{F(Q_1 > 0, Q_2 > 0)}{\text{mN}}$	$\frac{F(Q_1 < 0, Q_2 > 0)}{\text{mN}}$
4	3.41	–3.6
5	2.73	–2.95
6	2.40	2.49
7	1.94	–2.11
8	1.33	–1.56
9	0.95	–1.36
10	0.84	–0.96
15	0.41	–0.42
20	0.21	–0.17
25	0.11	–0.12

b) Measurement with various charges Q_1 and Q_2:

Table 2: The Coulomb force F acting on ball 2 as a function of its charge Q_2 (Q_2 >0, Q_1 = 36 nAs, d = 6 cm)

$\frac{U}{\text{kV}}$	$\frac{Q_2}{\text{nAs}}$	$\frac{F}{\text{mN}}$
5	7	0.32
10	14	0.91
15	22	1.4
20	28	2.01
25	36	2.76

Table 3: The Coulomb force F acting on ball 2 as a function of the charge Q_1 of ball 1 (Q_2 < 0, Q_2 = 36 nAs, d = 6 cm)

$\frac{U}{\text{kV}}$	$\frac{Q_1}{\text{nAs}}$	$\frac{F}{\text{mN}}$
–5	–7	–0.4
–10	–14	–0.96
–15	–22	–1.39
–20	–28	–2.1
–25	–36	–2.65

Evaluation and results

a) Measurement at various distances *d* between the balls:

Fig. 4 shows a graph of the measuring values of Table 1. The magnitude of the Coulomb force has a non-linear dependence on the distance d and is independent of the signs of the charges Q_1 and Q_2. If both charges have the same (opposite) sign, the Coulomb force is positive (negative).

In Fig. 5, the magnitudes of the forces are plotted against $1/d^2$. The straight line drawn through the origin agrees with the data points at small values of $1/d^2$. Thus, for large distances d the proportionality

$F \propto \frac{1}{d^2}$ is valid.

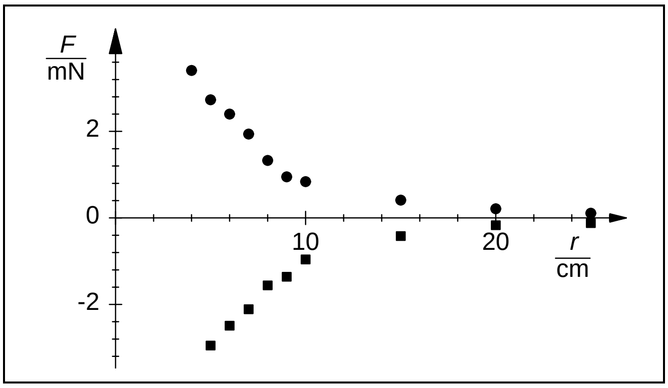
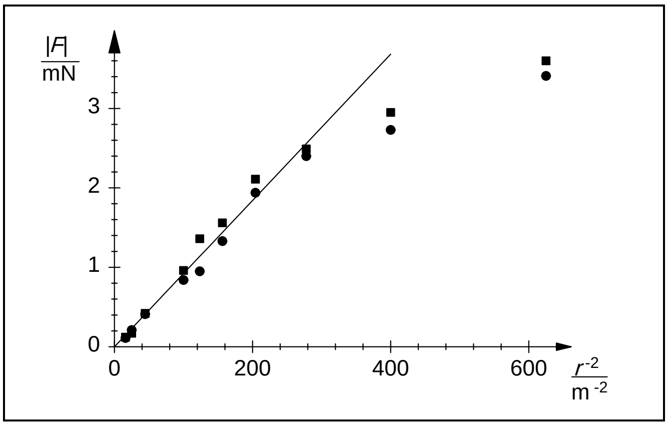


Fig. 4 The Coulomb force F between two charged balls as a function of the distance d between the balls
circles: measurement with equal charges
boxes: measurement with opposite charges

Fig. 5 The magnitude of the the Coulomb force F between two charged balls as a function of $1/d^2$
circles: measurement with equal charges
boxes: measurement with opposite charges



b) Measurement with various charges Q_1 and Q_2:

In Fig. 6, the measuring values of Tables 2 and 3 are summarized in one graph. The measuring values lie in a good approximation)) on a straight line through the origin. So the two proportionalities $F \propto Q_1$ and $F \propto Q_2$ are verified.

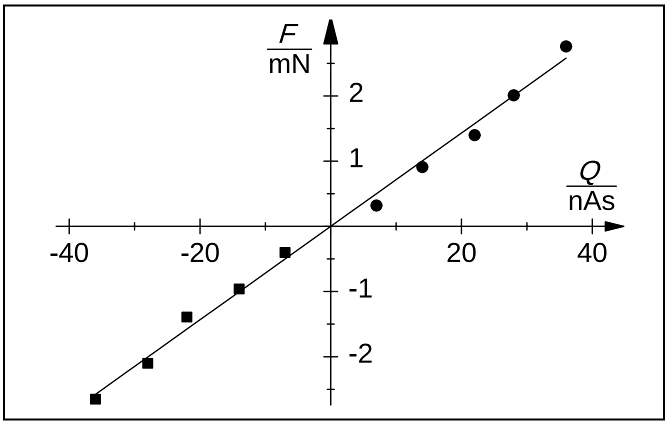


Fig. 6 The Coulomb force *F* acting on ball 2 at a fixed distance *d* = 6 cm
circles: measurement of *F* as a function of Q_2 (Q_1 = 36 nAs)
boxes: measurement of *F* as a function of Q_1 (Q_2 = 36 nAs)

c) Estimating the permittivity of free space:

Converting Eq. (I) leads to

$$\varepsilon_0 = \frac{\frac{1}{4\pi} \cdot \frac{Q_1}{d^2}}{\frac{F}{Q_2}}$$

The permittivity of free space can, therefore, be estimated from the slope of the straight line drawn through the origin in Fig. 5. The slope is

$$\frac{F}{Q_2} = 0.072 \, \frac{\text{mN}}{\text{nAs}}.$$

With the values Q_1 = 36 nAs and d = 0.06 m the result

$$\varepsilon_0 = 11 \cdot 10^{-12} \, \frac{\text{As}}{\text{Vm}}$$

is obtained.

The value quoted in the literature is:

$$\varepsilon_0 = 8.85 \cdot 10^{-12} \, \frac{\text{As}}{\text{Vm}}$$