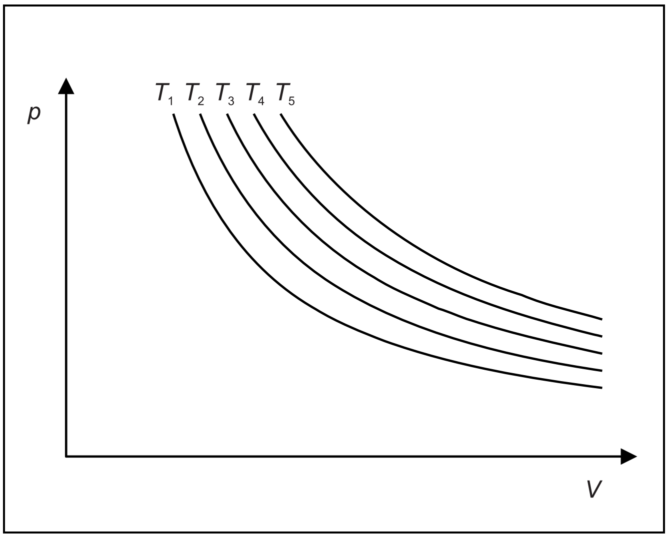
Pressure-dependency of the volume of a gas at a constant temperature (Boyle-Mariotte's law)

Objects of the experiments

- Measuring the volume V of an air column as a function of the pressure p at a constant temperature T.
- Confirming Boyle-Mariotte's law.

pV diagram of an ideal gas at a constant temperature *T*



Principles

The state of a quantity of ν moles of an ideal gas is completely described by the measurable quantities p (pressure), V (volume) and T (temperature). The relation between these three quantities is given by the perfect gas laws:

$$p \cdot V = \nu \cdot R \cdot T \quad \text{(I).}$$

R = 8.31 J K^{-1} mol^{-1}: gas constant

If p, V or T remains constant, then the other two quantities cannot be varied independently of each other. At a constant temperature, for example, Boyle-Mariotte's law states

$$p \cdot V = \text{const.} \quad \text{(II).}$$

This law is confirmed in the experiment by means of a gas thermometer. The gas thermometer consists of a glass capillary open at one end. A certain quantity of air is enclosed by means of a mercury seal. At an outside pressure p_0, the enclosed air has a volume V_0.

By pumping off air at room temperature with a hand pump, an underpressure Δp with respect to the outside pressure p_0 is generated at the open end of the capillary so that the pressure there is $p_0 + \Delta p$. The mercury seal itself exerts a pressure

$$p_{Hg} = \rho_{Hg} \cdot g \cdot h_{Hg} \quad \text{(III)}$$

ρ_{Hg} = 13.6 g cm^{-3}: density of mercury
g = 9.81 m s^{-2}: acceleration of free fall
h_{Hg}: height of the mercury seal

on the enclosed air so that the pressure of the enclosed air is

$$p = p_O + p_{Hg} + \Delta p \quad \text{(IV).}$$

The volume V of the enclosed air column is determined by the pressure p. V can be calculated from the height h of the air column and the cross- section of the capillary.

$$V = \pi \cdot \frac{d^2}{4} \cdot h \quad \text{(V)}$$

d = 2.7 mm: inside diameter of the capillary

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Apparatus

1 gas thermometer	382 00
1 hand vacuum and pressure pump	375 58
1 stand base, V-shape, 20 cm	300 02
1 stand rod, 47 cm	300 42
2 clamps with jaw clamp	301 11

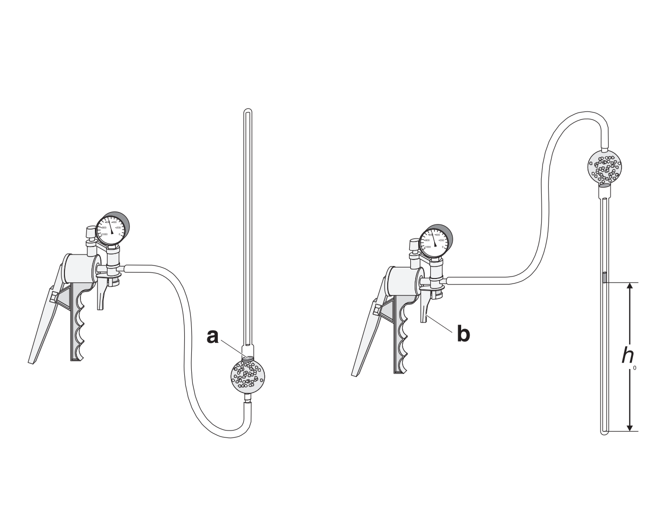
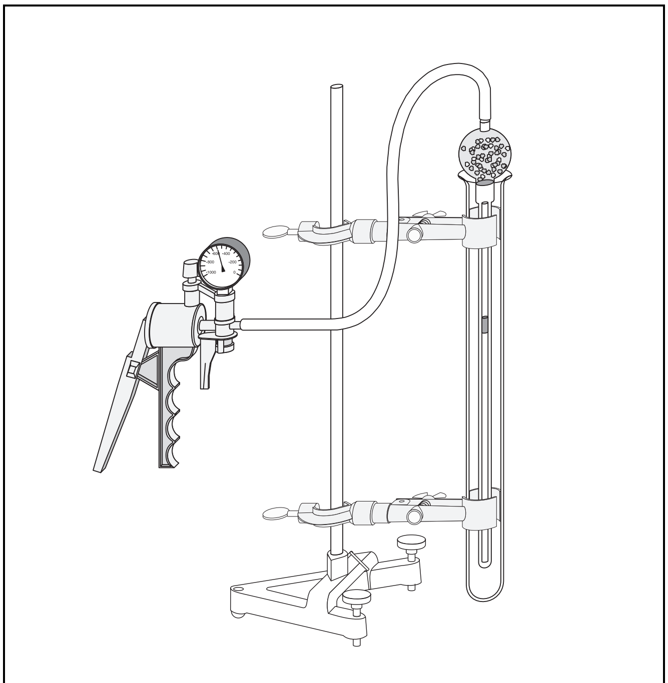


Fig. 1 Collecting the mercury globules and adjusting the initial volume V_0:

Fig. 2 Experimental setup for investigating the pressure-dependency of the gas volume at a constant temperature



Setup

Collecting the mercury globules:

– Connect the hand pump to the gas thermometer, and hold the thermometer so that its opening is directed downward (see Fig. 1).

– Generate maximum underpressure Δp with the hand pump, and collect the mercury in the bulge **(a)** so that it forms a drop.

The manometer of the pump displays the underpressure Δp as a negative value.

– If there are mercury globules left, move them into the bulge **(a)** by slightly tapping the capillary.

A small mercury globule which might have remained at the sealed end of the capillary will not affect the experiment.

Adjusting the gas volume V_0:

– Slowly turn the gas thermometer into its position for use (open end upward) so that the mercury moves to the inlet of the capillary.

– Open the ventilation valve **(b)** of the hand pump carefully and slowly to reduce the underpressure Δp to 0 so that the mercury slides down slowly as one connected seal.

– Mount the gas thermometer in the stand material.

If the mercury seal bursts due to strong ventilation or vibration:

– Recollect the mercury.

Carrying out the experiment

– Determine the outside pressure p_O.

– Read the height h_{Hg} of the mercury seal from the scale of the gas thermometer.

– Generate an underpressure Δp with the hand pump and increase it step by step.

– Each time read the height h of the air column, and record it together with Δp.

Measuring example

Outside pressure: $p_0 = 1011$ hPa

Height of the mercury seal: $h_{Hg} = 11$ mm

Table 1: The height h of the enclosed quantity of air as a function of the underpressure Δp.

$\frac{\Delta p}{\text{hPa}}$	$\frac{h}{\text{cm}}$
0	7.0
– 60	7.7
–100	8.0
–150	8.45
–200	8.9
–250	9.5
–300	10.5
–340	10.95
–410	12.1
–450	12.95
–500	14.1
–550	15.4
–600	17.15
–650	20.05
–690	22.5
–740	26.75
–780	31.35
–800	34.75

Evaluation

According to Eq (III) the pressure p_{Hg} exerted by the mercury seal is:

$$p_{Hg} = 13.6\,\frac{\text{g}}{\text{cm}^3} \cdot 9.81\,\frac{m}{s^2} \cdot 11\text{ mm} = 15\text{ hPa}$$

Table 2: The pressure p (calculated from the measuring values Δp of Table 1) of the enclosed quantity of air as a function of the volume V (calculated from the measuring values h of Table 1).

$\frac{V}{\text{mm}^3}$	$\frac{p}{\text{hPa}}$
401.1	1026
441.2	966
458.4	926
484.2	876
510	826
544.4	776
601.7	726
627.4	686
693.3	616
742	576
807.9	526
882.4	476
982.7	426
1148.9	376
1289.3	336
1532.8	286
1796.4	246
1991.2	226

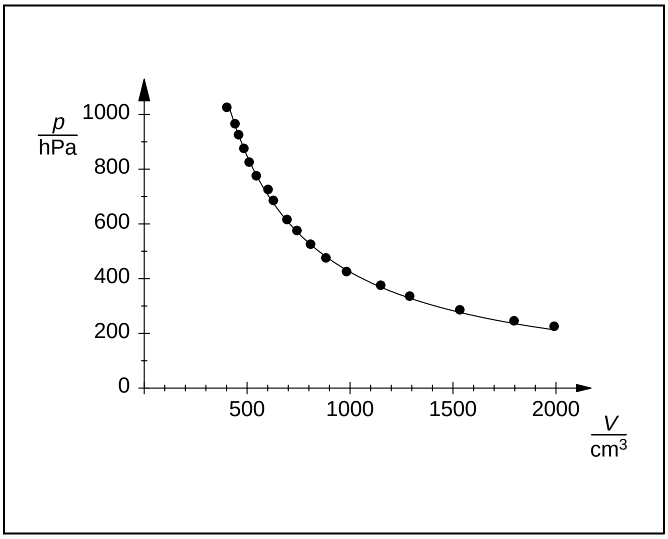


Fig. 3 The pressure p of the enclosed air column as a function of the volume V at a constant temperature T

Fig. 3 shows a plot of the measuring values of Table 2. The smooth curve drawn in is the hyperbola

$$p = \frac{C}{V}$$

with $C = 424\,000$ hPa mm^3.

Within the accuracy of measurement, this curve agrees with the measuring values. Eq. (II) is thus fulfilled for the enclosed air column, that is, the air column behaves as an ideal gas.

Results

At a constant temperature, the pressure and the volume of an ideal gas are inversely proportional to each other.

or:

The product of the pressure and the volume of an ideal gas is constant if the temperature is constant (Boyle-Mariotte's law).