Verifying the Bernoulli equation – measuring with a pressure sensor and CASSY

Objects of the experiment

- To verify the total pressure remains constant
- To verify that the product of the cross-section and the square root of the dynamic pressure is constant
- To verify the Bernoulli equation

Principles

Bernoulli's law states the relationship between static pressure p and flow velocity v, whereby the following applies to a frictionless, horizontally flowing stream through a stationary flow between two points labeled with indices 0 and 1 (Fig. 1):

$$p_0 + \frac{\rho}{2} v_0^2 = p_1 + \frac{\rho}{2} v_1^2 \qquad \text{(I)}$$

p_0, p_1: static pressure

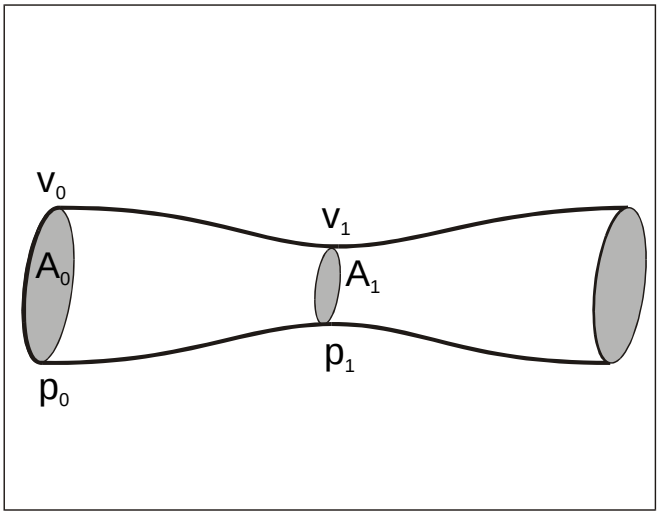
v_0, v_1: flow rate

ρ: density of the flow medium

In particular, equation (I) states that the total pressure p_{tot} has the same value everywhere:

$$p_1 + \frac{\rho}{2} v_1^2 = p_{tot} = \text{const} \qquad \text{(II)}$$

Fig. 1: Bernoulli equation schematically: cross sectional areas A_0 and A_1, flow velocity v_0 and v_1, static pressure p_0 and p_1



In the experiment described here, air flows through a wind tunnel whose cross-section decreases progressively in the direction of flow. Due to the incompressibility of air (which can always be assumed at the given flow rates) the flow velocities v_0 and v_1 at two different locations in the wind tunnel with cross-sectional areas A_0 and A_1 are given by the continuity equation:

$$v_0 A_0 = v_1 A_1 \qquad \text{(continuity equation)} \qquad \text{(III)}$$

The Bernoulli's equation (II) allows to eliminate v_1 in equation (III). By rearranging we obtain:

$$\sqrt{\Delta p} \cdot A_1 = \sqrt{\frac{\rho}{2}} \cdot v_0 \cdot A_0 \qquad \text{(IV)}$$

with

$$\Delta p = p_{tot} - p_1 \qquad \text{(V)}$$

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Apparatus

1 Wind tunnel 373 12
1 Suction and pressure fan 373 04
1 Pressure head 373 13
1 Measurement trolley 373 075

1 Sensor CASSY 524 010USB
or
1 Pocket CASSY 524 006
with
1 CASSY Lab 524 200
or
1 Mobile-CASSY 524 009

1 Pressure sensor S, ± 70 hPa 524 066

additionally required: 1 PC with Windows 98 or higher

The dynamic pressure Δp is determined by measuring the pressure difference. The cross-sections are stated at the various measuring points in the wind tunnel.

Safety notes

Mind the safety notes in the instruction sheet of the suction and pressure fan.

Before removing the protective grid or the nozzle:

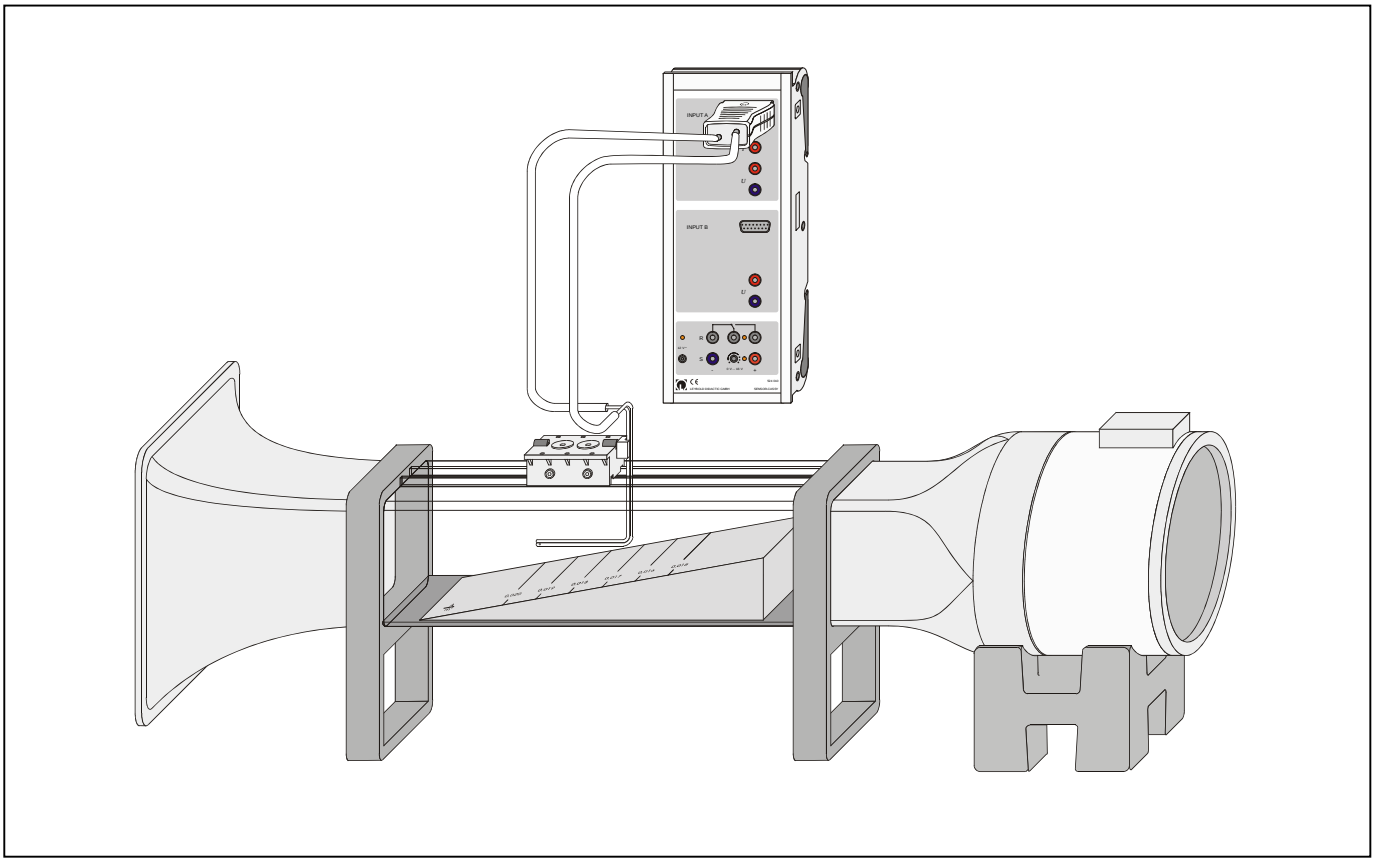
- pull out the mains plug
- wait for at least 30 seconds until the rotor comes to a complete stop.

Setup

Assemble the wind tunnel and the fan as shown in Fig 2. Insert in the fan into the outlet nozzle so that the air is drawn through the wind tunnel during the experiment. Ensure a clearance of approx. 1 m in front of the suction nozzle and behind the fan so that the air can be drawn into the wind tunnel without any turbulence.

- Mount the smoothing screen on the inlet.
- With the aid of the four screws secure the Bernoulli ramp underneath the plexiglass canopy so that the ramp height increases in the direction of flow.
- Mount the sealing strip (included in the equipment for the wind tunnel) on the sliding rails.
- Guide the pressure probe carefully bent section first, through the foam rubber seal of the sealing strip and secure the trolley.
- Push the probe down fully to the stop so that it reaches a position approx. 2 cm higher than the highest point of the ramp.
- Plug the pressure sensor S, ± 70 hPa to the Input A of the Sensor CASSY (Fig. 2) or to the Mobile CASSY (Fig. 4).
- Connect the total pressure probe by means of a hose to the nipple (left side) of the pressure sensor.
- Connect the static pressure probe by means of a hose to the nipple (right side) of the pressure sensor

Fig. 2: Experimental setup with Sensor CASSY schematically.



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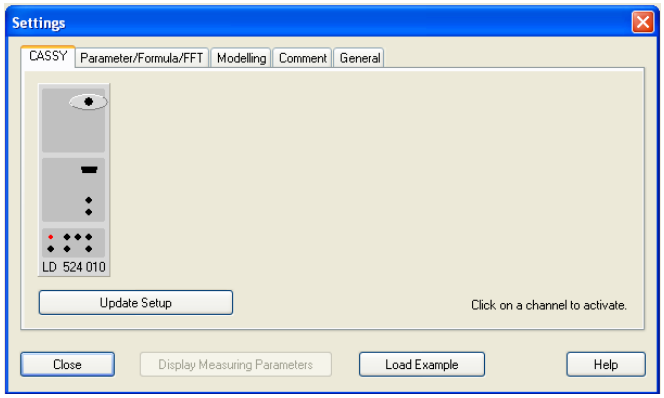
Carrying out the experiment

a) Measuring with Sensor CASSY

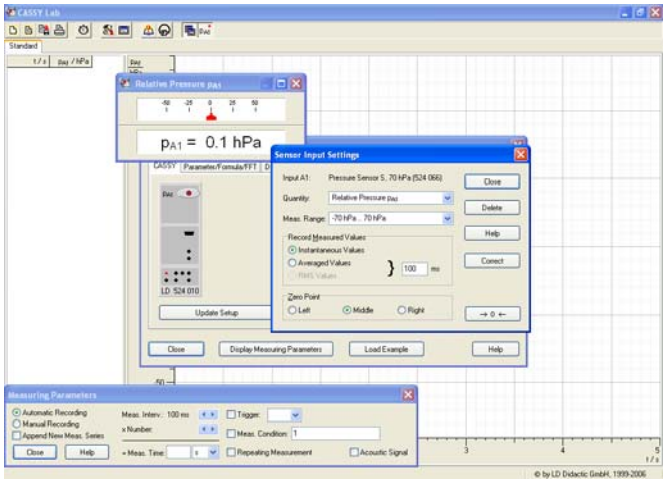
- If not yet installed install the software CASSY Lab and open the software.
- Open the window “Settings” using the tool box button or function key F5 from the top button bar:

- Sensor CASSY with the connected pressure sensor S at “Input A” should be displayed at the tab “CASSY” if Sensor CASSY is connected via the USB port to the computer.

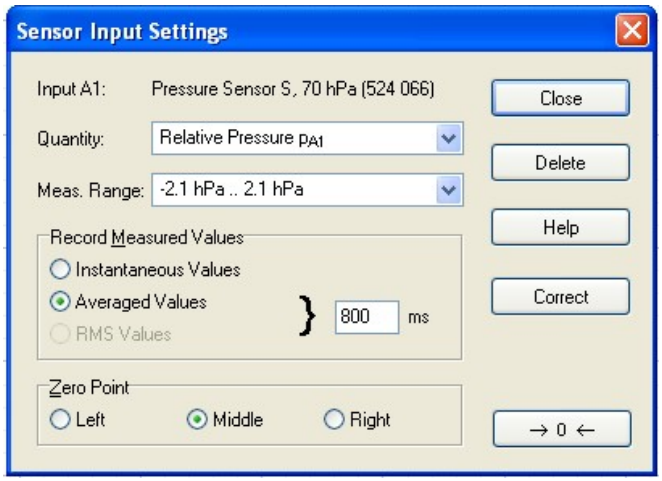


- Activate the connected sensor at Input A by clicking at the pressure sensor S.

Note: Further details about connecting sensors to Sensor CASSY can be found in the manual “CASSY Lab Quick Start”.

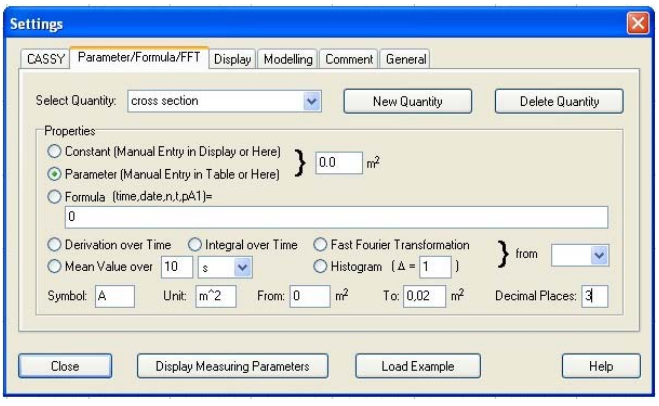


- In the window “Sensor Input Settings” select “Averaged Values” and enter the value, e.g. “800 ms”.
- Select the measurement range to “-2.1 hPa … 2.1 hPa”.
- Disconnect the pressure sensor S from the pressure probe.
- Set the pressure sensor by clicking in the window “Sensor Input Settings” at the button “-> 0 <-“.



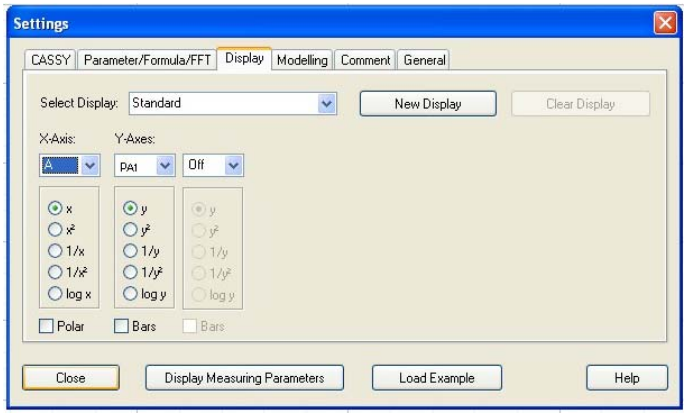
- Connect the pressure sensor S to the pressure probe.
- Set the blower to its minimum speed (i.e. left limit position of blower control) and only then switch it on.
- Slowly increase speed of the air blower until the desired speed is reached.
- Click on the button and select “Manual Measurement”.

- To define a parameter A for the cross-section in CASSY Lab use the tool box button or function key F5 to open the window “Settings”. Select the tab “Parameter/Formula/FFT”
- Select new Quantity and enter the name “cross section”.



- Enter the symbol “A” and the “m^2”.
- Enter appropriate values for the range, e.g. “0.014” m/s^2 and “0.022” m/s^2 and enter “3” for decimal places.
- After the “new quantity” cross section the “standard display” can be changed to plot the pressure difference (i.e. dynamic pressure) as function of the cross section A.

- To change the standard display in CASSY Lab use the tool box button or function key F5 to open the window “Settings”. Select the tab “Display”.



- Select the quantity cross section A as x-axis (and the pressure pA1 as y-axis).
- Place the measurement trolley at with end of the pressure head at the position F.
- To record a value click on the button or use the function key F9.
- Enter the corresponding cross section 0.015 m^2 in column A of the table (left side of CASSY Lab window).
- Repeat the measurement for the positions “A” to “E”.

Note:

The measurement may be repeated for different fan speeds.

To apply the rest button “-> 0 <-“ for the pressure sensor the window “Sensor Input Settings” have to be invoked if not open. This can be achieved e.g. by clicking the right mouse button on the pressure button in the top menu bar.

b) Measuring with Mobile CASSY

- Switch on the Mobile CASSY with the MENU key.
- Call the main menu using the MENU key.
- Select the submenu “Quantities” by using the arrow keys or and enter the submenu using the right key.
- Go to the submenu “p” using the right key.
- Chose “Compensate Offset” and set the pressure to zero by pushing the right key.
- Press MENU key and then the left key to display the measured pressure value.
- Place the measurement trolley at with end of the pressure head at the position F.
- Read off the pressure difference.
- Repeat this measurement procedure several times and calculate the mean average. It is recommended to compensate the offset before each measurement.
- Repeat the measurement for the positions “A” to “E”.

Note: For further hints using Mobile CASSY refer to the instruction sheet 524 009.

Measuring example

a) Measuring with Sensor CASSY

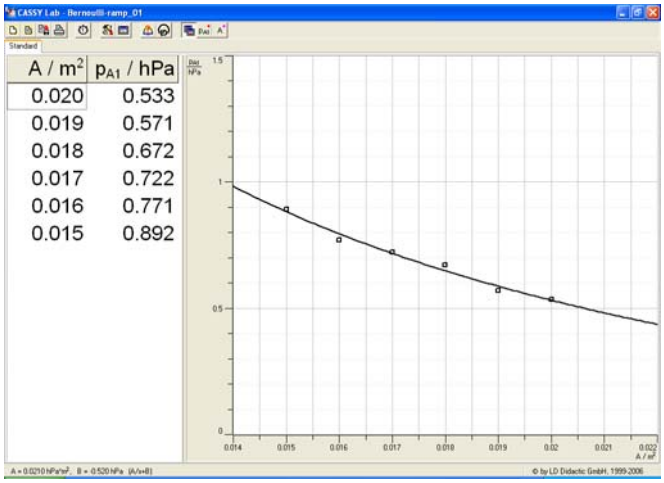


Fig. 2: Pressure difference as function of cross section A. The solid line corresponds to a fit of a hyperbola.

b) Measuring with Mobile CASSY

Table 1: Pressure difference Δp (mean average over 5 measurements) at the positions A to F.

position	$\frac{A}{m^2}$	$\frac{\Delta p}{hPa}$
A	0.020	0.533
B	0.019	0.571
C	0.018	0.672
D	0.017	0.722
E	0.016	0.771

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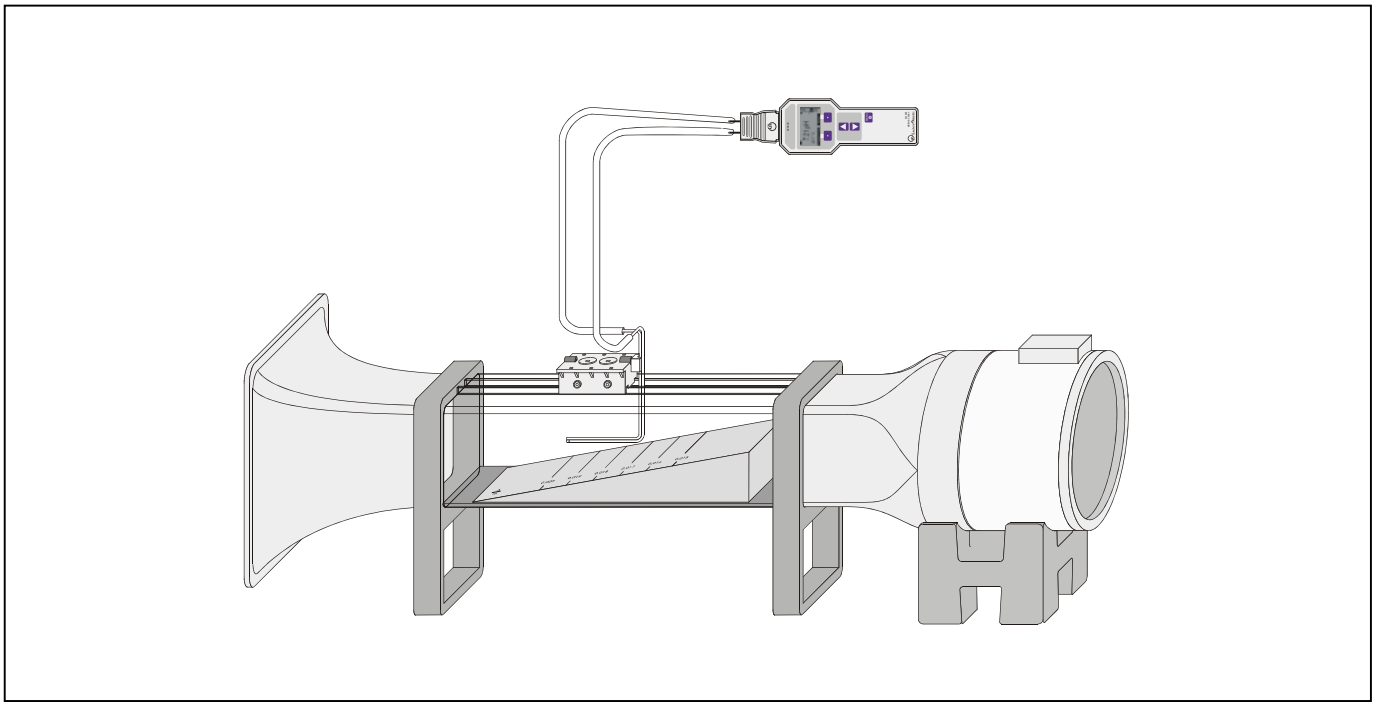
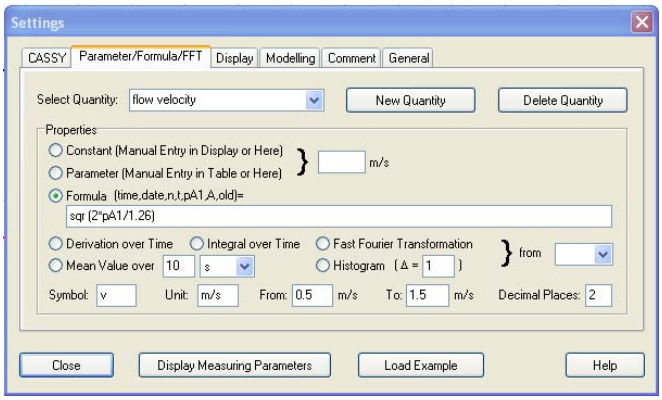


Fig. 4: Experimental setup with Mobile CASSY schematically.

Evaluation and results

a) Measuring with Sensor CASSY

- To define a parameter for the flow velocity use the tool box button or function key F5 to open the window “Settings”. Select the tab “Parameter/Formula/FFT”
- Select new Quantity and enter the name “flow velocity”.



- Select “formula” and enter the formula for the flow velocity “sqr(2 * pA1/1.26)”
- Enter the symbol “v” and the “m/s”.
- Enter appropriate values for the range, e.g. “0.5” m/s^2 and “1.5” m/s^2 and enter “2” for decimal places.
- For plotting v as a function of A define a new display. Click on the tool box button or function key F5 to open the window “Settings”. Select the tab “Display”.
- Click on the button “New Display” and enter a name, e.g. “flow velocity”.
- Select the quantity “A” as x-axis and the pressure pA1 as y-axis.

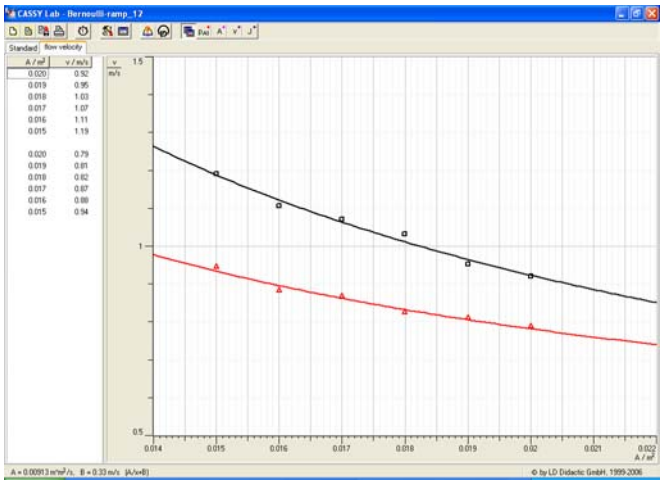
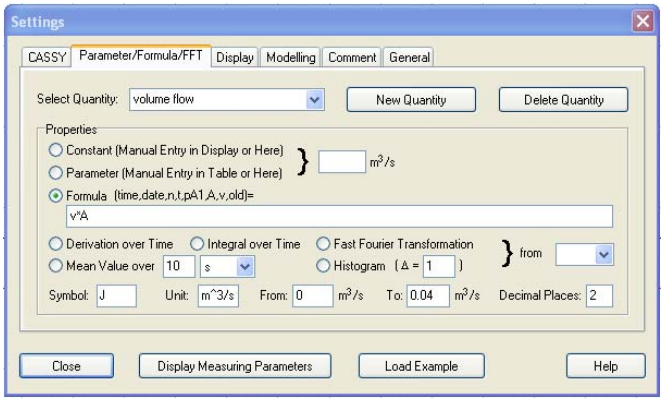


Fig. 5: Flow velocity v as a function of the cross section A for two different speeds of the fan. The solid lines correspond to a fit of hyperbolas in accordance with equation (IV).

- To define a parameter for volume flow use the tool box button or function key F5 to open the window “Settings”. Select the tab “Parameter/Formula/FFT”
- Select new Quantity and enter the name “volume flow”.



- Select “formula” and enter the formula “v*A”
- Enter the symbol “J” and the “m^3/s”.
- Enter appropriate values for the range, e.g. “0” m/s^2 and “0.4” m/s^2 and enter “2” for decimal places.

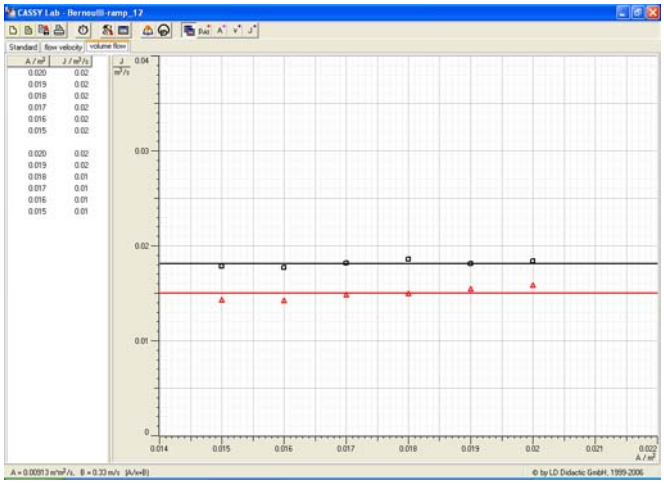


Fig. 6: Volume flow J as a function of the cross section A for two different speeds of the fan. The solid lines correspond to the mean average in accordance with continuity equation (III).

b) Measuring with Mobile CASSY

Table 2: Flow velocity v and volume flow J evaluated from the pressure difference Δp of table 1 at the positions A to F.

position	$\frac{v}{m/s}$	$\frac{J}{m^3/s}$
A	0.92	0.018
B	0.95	0.018
C	1.03	0.019
D	1.07	0.018
E	1.11	0.018

The flow velocity $v = \sqrt{\frac{2 \cdot \Delta p}{\rho}}$ increases with decreasing cross-section A. The volume flow $J = v \cdot A$ is constant over the entire decreasing cross-sectional area. Thus the predictions based on the Bernoulli’s equation (I) are verified qualitatively.

Supplementary information

Additionally, the total pressure can be measured along the progressively decreasing cross section. Therefore the hose is connected to the total pressure head only as shown in Fig. 7.

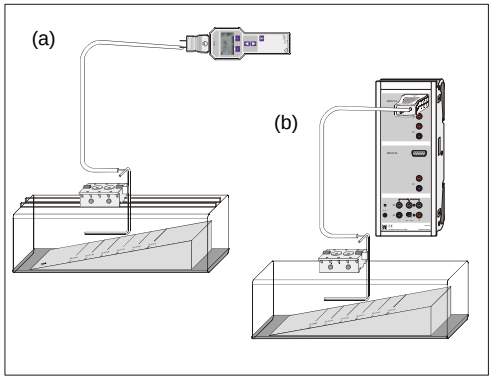


Fig. 7: Experimental setup for measuring the total pressure with Mobile CASSY (a) or Sensor CASSY (b).

The result of the measurement is depicted in Fig. 8. The measurement verifies that the total pressure remains constant over the entire measurement section.

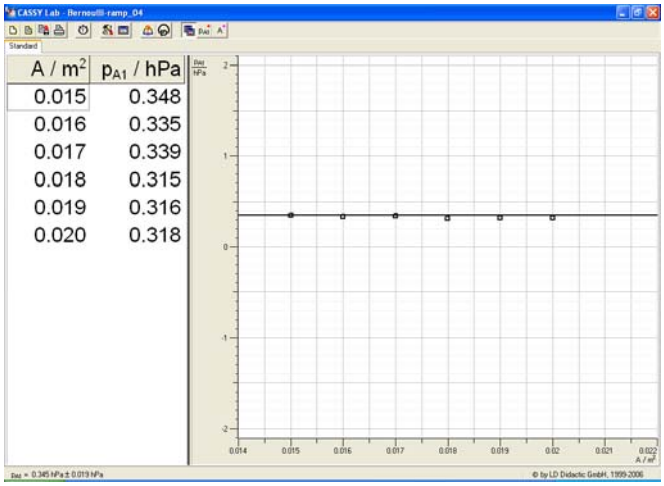


Fig. 8: Total pressure as function of the cross section A. The solid line corresponds to the mean value.

When the static pressure head is connected to the pressure sensor S:

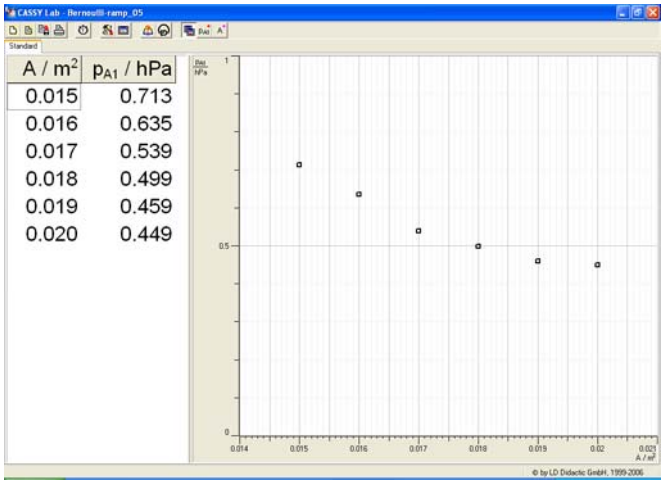


Fig. 9: Static pressure as function of the cross section A. The static pressure increases with decreasing cross section.