Assembling a falling-ball viscosimeter to determine the viscosity of viscous fluids

Objects of the experiments

- Assembling a falling-ball viscosimeter.
- Determining the viscosity of glycerine.

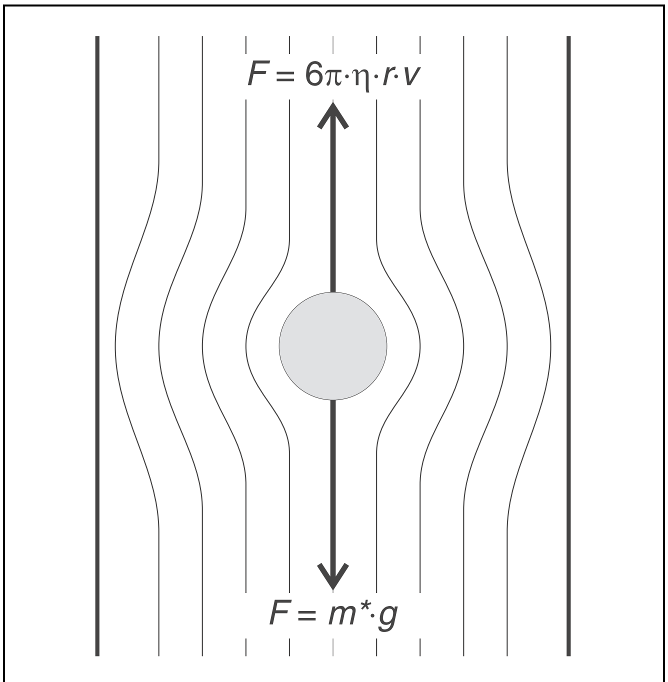
Principles

A body moving in a fluid is acted on by a frictional force in the opposite direction of its velocity. The magnitude of this force depends on the geometry of the body, its velocity, and the internal friction of the fluid. A measure for the internal friction is given by the dynamic viscosity η. For a sphere of radius r moving at velocity v in an infinitely extended fluid of dynamic viscosity η, *G. G. Stokes* derived the frictional force

$$F_1 = 6\pi \cdot \eta \cdot v \cdot r \qquad \text{(I).}$$

If the sphere falls down vertically in the fluid, it will move at a constant velocity v after a certain time, and there will be an equilibrium between all forces acting on the sphere: the frictional force F_1, which acts upward, the buoyancy force

$$F_2 = \frac{4\pi}{3} \cdot r^3 \cdot \rho_1 \cdot g \qquad \text{(II),}$$



which acts upward too, and the downward acting gravitational force

$$F_3 = \frac{4\pi}{3} \cdot r^3 \cdot \rho_2 \cdot g \qquad \text{(III)}$$

ρ_1: density of the fluid
ρ_2: density of the sphere
g: acceleration of free fall

These forces fulfil the relation

$$F_1 + F_2 = F_3 \qquad \text{(IV)}$$

The viscosity can, therefore, be determined by measuring the rate of fall v.

$$\eta = \frac{2}{9} \cdot r^2 \cdot \frac{(\rho_2 - \rho_1) \cdot g}{v} \qquad \text{(V)}$$

where v is to be determined from the distance s and the time t of fall. The viscosity then is

$$\eta = \frac{2}{9} \cdot r^2 \cdot \frac{(\rho_2 - \rho_1) \cdot g \cdot t}{s} \qquad \text{(VI)}$$

In practice, Eq. (I) has to be corrected since the assumption of an infinitely extended fluid is unrealistic and the velocity distribution of the fluid particles with respect to the surface of the sphere is influenced by the finite dimensions of the fluid. For the movement of the sphere along the axis of a fluid cylinder of radius R and infinite length, for example, the frictional force is

$$F_1 = 6\pi \cdot \eta \cdot r \cdot v \cdot \left(1 + 2.4 \cdot \frac{r}{R}\right) \qquad \text{(VII)}$$

Eq. (V) thus is changed into

$$\eta = \frac{2}{9} \cdot r^2 \cdot \frac{(\rho_2 - \rho_1) \cdot g \cdot t}{s} \cdot \frac{1}{1 + 2.4 \cdot \frac{r}{R}} \qquad \text{(VIII)}$$

If the finite length L of the fluid cylinder is taken into account, there are further corrections of the order $\frac{r}{L}$.

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Apparatus

1 steel ball, 16 mm dia.	200 67 288
1 guinea-and-feather apparatus	379 001
6 glycerine, 99 %, 250 ml	672 121
1 counter P	575 45
1 holding magnet with clamp	336 21
1 low-voltage power supply, 3, 6 ,9 ,12 V	522 16
1 morse key	504 52
1 stand base, V-shape	300 01
1 stand rod, 100 cm	300 44
1 stand rod, 25 cm	300 41
1 Leybold multiclamp	301 01
1 clamp with jaw clamp	301 11
1 steel tape measure, 2 m	311 77
1 pair of magnets, cylindrical	510 48
connection leads	
additionally recommended:	
1 precision vernier callipers	311 54
1 measuring cylinder, 100 ml, plastic	590 08
1 electronic balance LS 200, 200 g: 0,1 g	667 793

Setup

The experimental setup is illustrated in Fig. 1:

– Assemble the stand material.
– Hold the guinea-and-feather apparatus inclined, and fill the glycerine in slowly, if possible without bubbles, almost up to the top.

Note:

Air bubbles in the fluid influence the viscosity and the density. If there are small air bubbles in the fluid after the filling, wait a few hours before carrying out the experiment.

– Fix the guinea-and-feather apparatus in the clamp with jaw clamp **(c)** so that it is propped up on the experiment table.
– Turn the knurled screw **(a)** of the holding magnet down until stopping, so that the iron core **(b)** sticks out of the coil former.
– Connect the holding magnet to the DC output of the low-voltage power supply with the morse key in the line coming from the negative pole so that the connection is closed when the morse key is in the rest position.
– Supply an output voltage of 12 V, and hang the steel ball up on the iron core **(b)**.
– Turn the knurled screw **(a)** upward by about five turns.
– Position the holding magnet with the steel ball above the fluid column in a way that the steel ball is on concenter with the cylinder axis and completely dipped in.
– Make a mark at the guinea-and-feather apparatus some centimetres above its bottom and measure the distance of fall *s* between the lower edge of the ball and the mark.

Connection of the counter P:

– Connect the ground socket of the counter P to the feeder socket **(d)** of the morse key, the start input to socket **(e)**, and the stop input to socket **(f)**.
– Choose the measuring range ms.

Carrying out the experiment

– Set the counter P to zero by pressing the key “0”.
– Trigger off the morse key, and observe the falling ball.
– As soon as the ball has reached the mark **(c)**, release the morse key.
– Read the time of fall *t* from the counter P and record it.

If the ball does not fall at all or if it falls with a delay:

– Check the connections.
– Turn the iron core a bit upward.
– Choose a lower voltage for the holding magnet.

If the ball falls without the morse key‘s being triggered:

– Turn the iron core a bit downward.

Repeating the measurement:

– Turn the voltage for the holding magnet to 12 V and turn the knurled screw **(a)** to stop.
– Get grip of the steel ball from outside on the bottom of the vessel with the pair of magnets sticking together (red mark outward), and move the ball slowly upward along the wall of the vessel until it reaches the holding magnet. Using a bent piece of wire, for example, push the ball exactly below the iron core (see Fig. 2).
– Turn the knurled screw upward again, set the counter P to zero, and repeat the measurement of the time of fall.

If the devices recommended in addition are available (see above):

– Determine the inner diameter D of the guinea-and-feather apparatus, and the diameter *d* and the mass m_2 of the steel ball.
– Put the measuring cylinder on the electronic balance, and counterbalance.
– Fill 100 ml of glycerine from the storage bottle into the measuring cylinder, and determine its mass.

Measuring example

Table 1: times of fall *t*

n	$\frac{t}{ms}$
1	2084
2	2110
3	2104
4	2036
5	2116

distance of fall: s = 66.6 cm

diameter of the ball: d = 16.0 mm

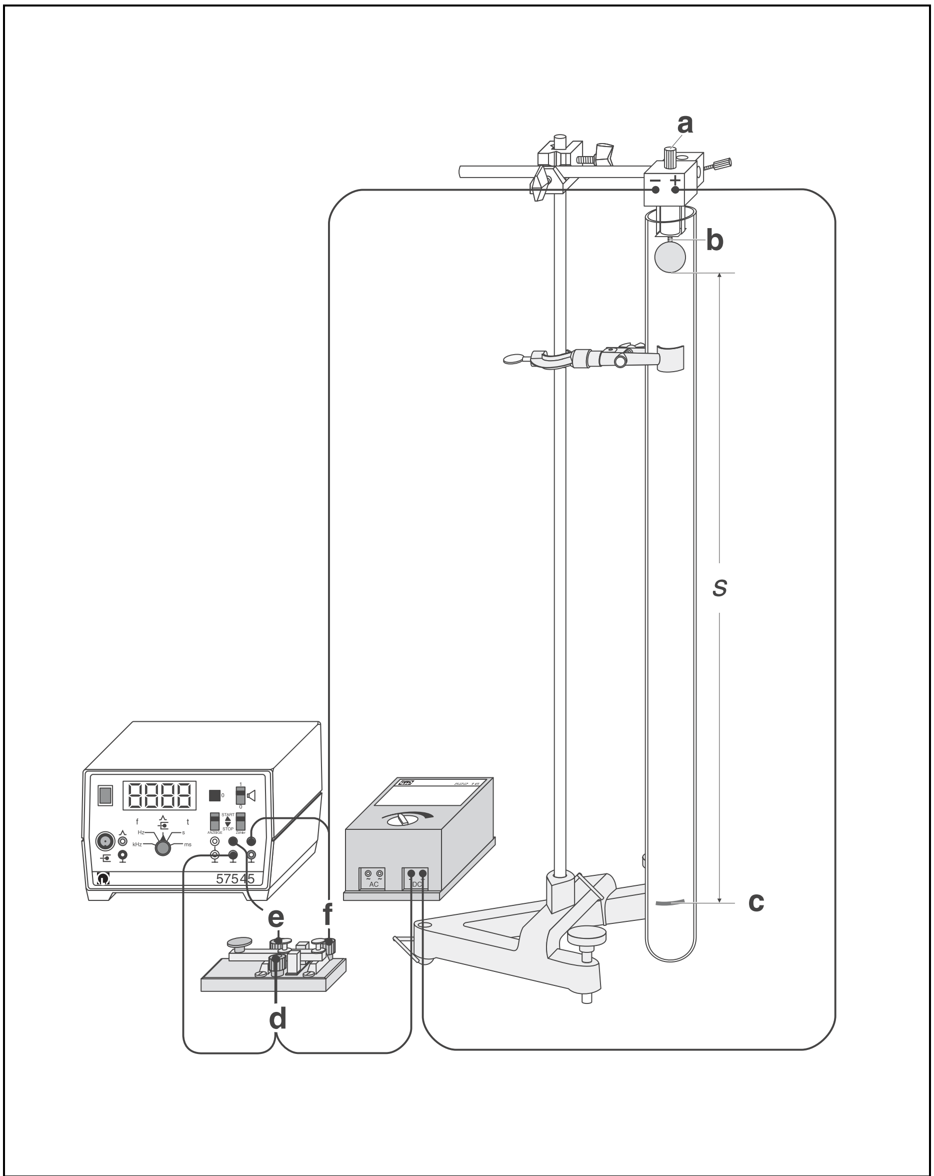
diameter of the guinea-and-feather apparatus: D = 44 mm

mass of the ball: m_2 = 16.7 g

mass of 100 ml of glycerine: m_1 = 125.4 g

*) If these quantities are not measured, use the following values for the further evaluation:

r = 8 mm, R = 22 mm, ρ_1 = 1260 kg m^{-3}, ρ_2 = 7790 kg m^{-3}



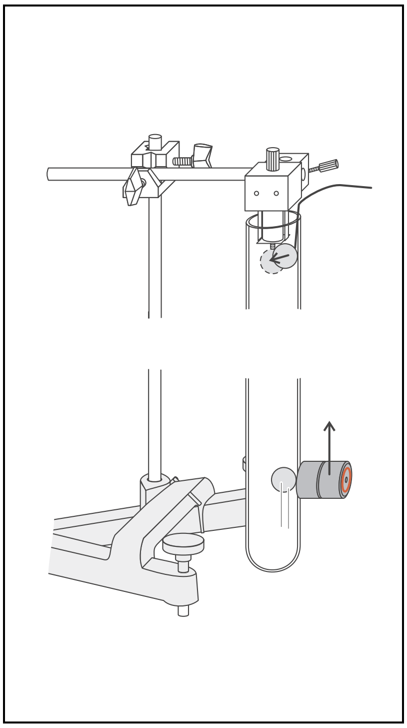


Fig. 2 Returning the steel ball ▲

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Fig. 1 Experimental setup for the determination of the viscosity of glycerine

Evaluation and results

Time of fall:

Mean value of the measuring results of Table 1: t = 2.090 s

Density of the ball:

From the measuring results you find

$V_2 = 2.14\ \text{cm}^3$ and $\rho_2 = \frac{m_2}{V_2} = 7787\ \text{kg m}^{-3}$

Density of glycerine:

$\rho_1 = 1254\ \text{kg m}^{-3}$

According to Eq. (VIII), the viscosity thus is:

$\eta = 1.53\ \text{kg m}^{-1}\ \text{s}^{-1}$

The value quoted in the literature is (ϑ = 20 °C):

$\eta = 1.480\ \text{kg m}^{-1}\ \text{s}^{-1}$

When you compare your result with the value taken from the literature, keep in mind that the viscosity of glycerine strongly depends on the temperature.

Supplementary information

After application of Eqs. (II), (III) and (VII), the equation of motion of the falling ball

$$m \cdot \frac{dv}{dt} = F_3 - F_2 - F_1$$

can be converted into the following differential equation:

$$\frac{dv}{dt} = \frac{\rho_2 - \rho_1}{\rho_2} \cdot g - \frac{v}{\tau}$$

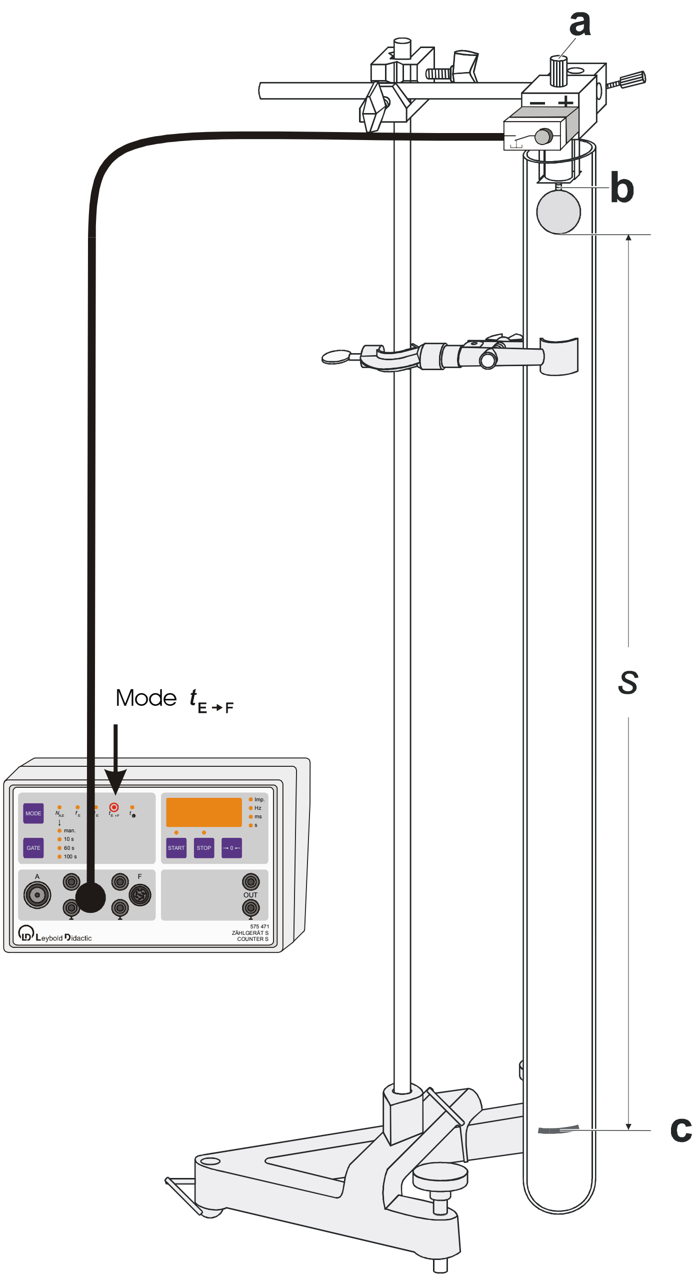
with the time constant $\tau = \frac{2}{9} \cdot \frac{r^2 \cdot \rho_2}{\eta} \cdot \frac{1}{1 + 2.4 \cdot \frac{r}{R}}$

With the initial condition $v(t = 0) = 0$, its solution is:

$$v = \frac{2}{9} \cdot \frac{r^2 \cdot (\rho_2 - \rho_1) \cdot g}{\eta} \cdot \frac{1}{1 + 2.4 \cdot \frac{r}{R}} \cdot \left(1 - e^{-\frac{t}{\tau}}\right)$$

With the parameters of our measurement you find τ = 39 ms; the total time of fall is 2.1 s. The assumption of a constant rate of fall thus turns out to be a justified approximation.

Wiring diagram (schematically) for alternative equipment list P1.8.3.1, i.e. measuring with counter S (575 471) and Holding magnet adapter with a release mechanism (336 25, i.e. 570 10 with multi-core cable).



Carrying out the experiment

- Select Measurement Mode $t_{E\to F}$.
- Set the counter S to zero by pressing the key “→0←”.
- Press the key “START”.
- Start measurement by pressing the button of the switch attached to the holding magnet.
- As soon as the ball has reached the mark (c), Press the “STOP” key at the counter S.

For further details refer to leaflet P1.8.3.1