Mechanics

Oscillations
Torsion pendulum

LD
Physics
Leaflets

P1.5.3.3

Free rotational oscillations

Recording and evaluating with CASSY

Experiment Objectives

■ Recording a torsion pendulum's amplitude as a function of time.

■ Determining the damping coefficients.

■ Studying the transition from underdamping to critical damping and to overdamping.

Fundamentals

Oscillations and waves are very important both in nature and in technology. The study of related phenomena therefore requires both the experimental and the theoretical angles. This helps understand the fundamental models and laws of physics.

Rotational oscillations represent a special case of mechanical oscillations. Yet all major phenomena can be studied on them.

This experiment studies free rotational oscillations at different damping states.

The physical value that fully describes the system's state at the given time t is the angle of deflection $\varphi(t)$ from the rest position (where $\varphi = 0$).

The spiral spring's effect on the torsion pendulum is given by Hooke's law:

$$M_f = D \cdot \varphi(t)$$

where D is the spring constant and M_f the torque on the torsion pendulum resulting from the spring.

In addition, the eddy-current brake exerts a torque on the pendulum:

$$M_r = k \cdot \dot{\varphi}(t)$$

where k is the constant of friction and $\dot{\varphi}(t)$ the first time derivative of the angle of deflection, so the angular velocity.
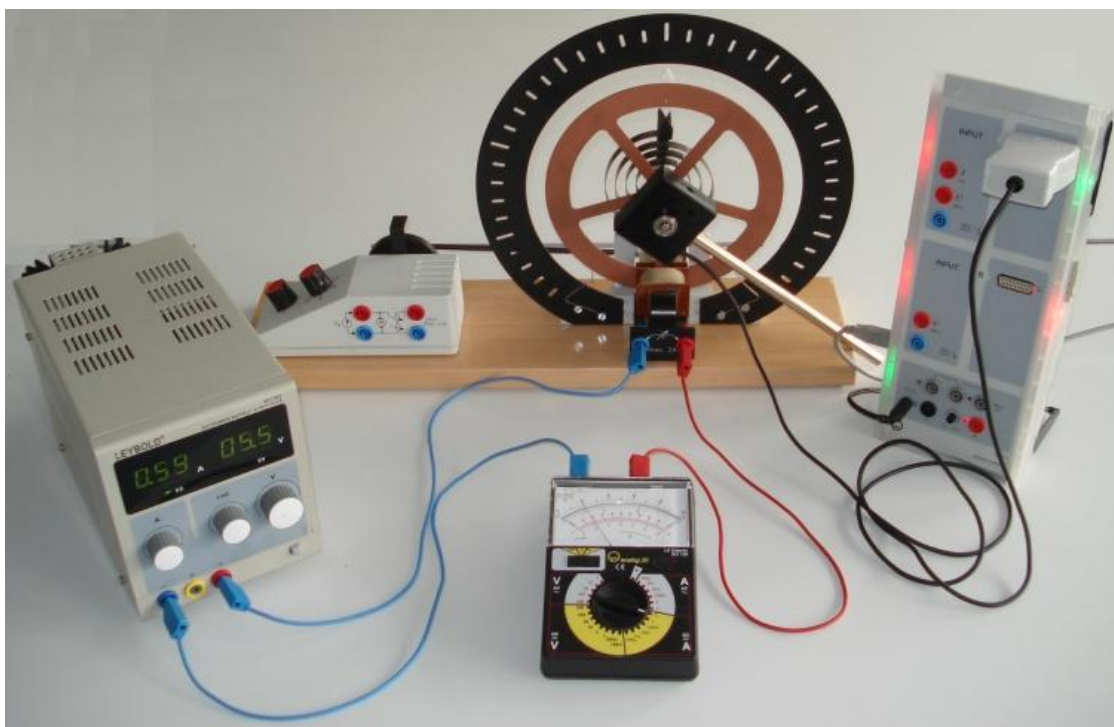

Fig 1: Experiment setup for free rotational oscillations with damping

Equipment

1	Torsion pendulum 346 00
1	DC Power Supply 0…16 V, 0…5 A 521 545
1	Sensor-CASSY 2 .... 524 013
1	Rotary motion sensor S 524 082
1	CASSY Lab 2 .... 524 220
1	Multimeter LDanalog 20 .... 531 120
1	Connecting lead, 100 cm, blue 500 442
1	Connecting lead, 100 cm, red/blue, pair 501 46
1	PC with Windows XP/Vista/7/8

Safety note

Pay attention to the maximum current on the electric magnet for the eddy-current brake:

$$I_{max} = 1 \text{ A (briefly 2 A)}$$

The sum of the two torques adds up to the negative total torque (since it is reversed):

$$- M_g = M_f + M_r$$

for which, per Newton:

$$M_g = I \cdot \ddot{\varphi}(t)$$

where I is the torsion pendulum's moment of inertia and $\ddot{\varphi}(t)$ the angular acceleration.

Hence:

$$I \cdot \ddot{\varphi}(t) + k \cdot \dot{\varphi}(t) + D \cdot \varphi(t) = 0. \qquad (1)$$

Equation (1) is the equation of motion describing the free, damped oscillation. It is thereby a second-order ordinary, homogeneous, linear differential equation with a unique, known solution.

Introduce the following values to express the formulas more clearly:

- Damping coefficient

$$\delta = \frac{k}{2\,I}$$

- Natural frequency of the undamped torsion pendulum

$$\omega_0 = \sqrt{\frac{D}{I}}$$

- Frequency of the damped torsion pendulum

$$\omega = \sqrt{\omega_0{}^2 - \delta^2}$$

(exists only for $\omega_0 > \delta$).

Mathematically as well as physically, the following three cases can be distinguished:

Underdamped $\delta < \omega_0$

The damping is low. In this case, the general solution to equation (1) goes

$$\varphi(t) = e^{-\delta t} \cdot (A \cdot \cos \omega t + B \cdot \sin \omega t). \qquad (2)$$

Constants A and B are defined by the values specified for the launching angle $\varphi(0) = \varphi_0$ and the launching angle velocity $\dot{\varphi}(0) = \dot{\varphi}_0$, thus producing

$$A = \varphi_0 \text{ and } B = \left[\frac{\dot{\varphi}_0 + \varphi_0 \cdot \delta}{\omega} \right].$$

From equation (2) we can say that the amplitude decreases with time by a factor of $e^{-\delta t}$, i.e. it is halved after the half-life $T_{1/2} = \frac{\ln 2}{\delta}$ and that the damping ratio of two consecutive amplitudes is constant, $q = \frac{\varphi_{n+1}}{\varphi_n} = e^{-\delta T}$.

Thereby $T = \frac{2\pi}{\omega}$ is the period of the damped oscillation. The exponent

$$\Lambda = \delta \cdot T = \ln \frac{\varphi_n}{\varphi_{n+1}} = \ln \frac{1}{q}$$

is designated as a logarithmic damping decrement.

Critically damped (or aperiodic boundary case) $\omega_0 = \delta$

If the damping is large enough, the pendulum quickly moves to the rest position from the deflected state without crossing the rest position. In this case, the solution to equation (1) goes

$$\varphi(t) = e^{-\delta t} \cdot (A + B \cdot t) \qquad (3)$$

with constants

$$A = \varphi_0 \text{ and } B = \dot{\varphi}_0 + \varphi_0 \cdot \delta .$$

This is no longer a periodic curve, so the solution (3) does not feature the angular frequency ω (therefore the designation "aperiodic"). Out of all the aperiodic cases (including overdamped, below), the pendulum takes the least time to reach the rest position with the lowest damping (therefore the designation "boundary case").

Overdamped $\delta > \omega_0$

With very high damping, the pendulum slowly approaches the rest position asymptotically. In this case, the general solution to equation (1) goes

$$\varphi(t) = e^{-\delta t} \cdot \left(A \cdot e^{\lambda t} + B \cdot e^{-\lambda t} \right) \qquad (4)$$

with $\lambda = \sqrt{\delta^2 - \omega_0{}^2}$.

Experiment setup

- Set up the experiment according to Fig. 1.

- Screw the stand rod into the rotary motion sensor.

- Carefully insert the axis of the rotary motion sensor S into the provided socket on the pendulum (see Fig. 2, left). Do this without holding the pendulum, to avoid potential imbalance. The O-ring must be fully inserted on the axis of the rotary motion sensor S for a nonslip connection of the two axes (Fig. 2, right).

Fig. 2: Mounting of the rotary motion sensor S onto the torsion pendulum

- Carefully lay the stand rod of the rotary motion sensor S on the desk (see Fig. 3) so the two axes are in a straight extension without a mechanical load.
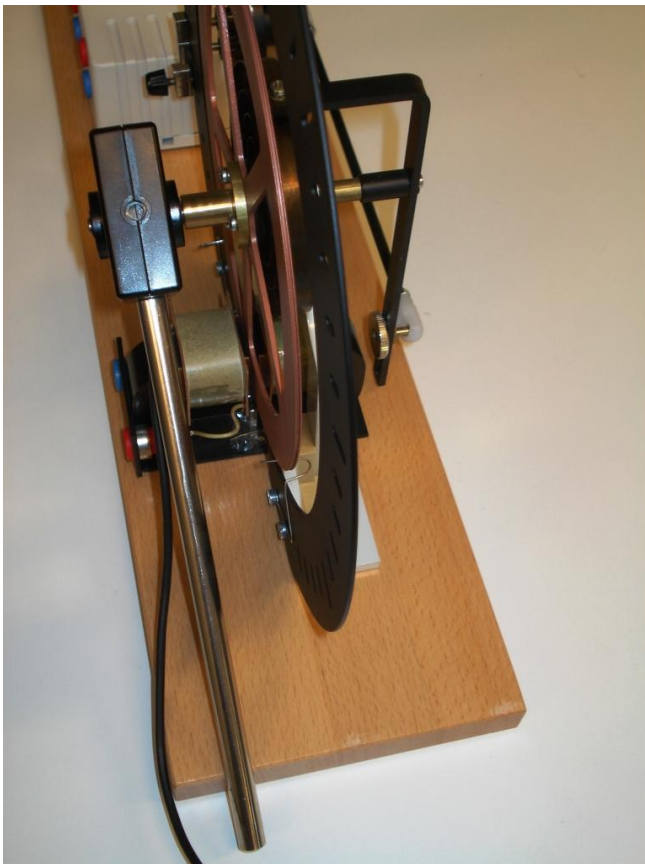
Fig. 3: Rotary motion sensor S on the torsion pendulum

- Connect the rotary motion sensor S to the Sensor-CASSY 2.

- Connect the power supply and the multimeter to the electric magnet for the eddy-current brake according to Fig. 4.

- Do not turn on the power supply yet!

- Set the torsion pendulum so the phase position indicators of the excitation and of the pendulum's body point toward each other. If necessary, slightly turn the excitation motor's drive wheel.
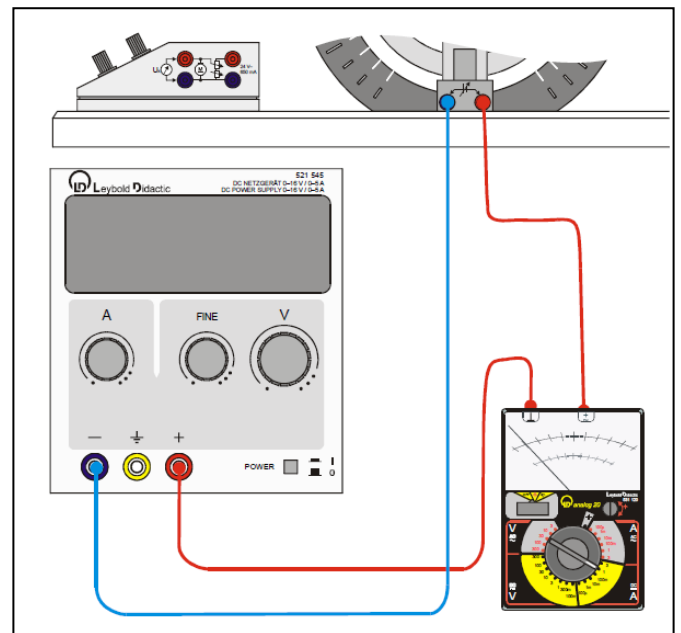

Fig. 4: Connection of the eddy-current brake

Carrying out the Experiment

> **Safety note**
>
> Pay attention to the maximum current on the electric magnet for the eddy-current brake:
>
> $$I_{max} = 1 \text{ A (briefly 2 A)}$$

a) Underdamped / Determining the damping coefficients

- Load the settings in CASSY Lab 2. Do not start the measurement yet!

- Turn on the power supply.

- Observe the current on the measuring instrument. Set and write down a current of approx. 0.3 A.

- If the pendulum is at rest, calibrate the angle of deflection α_{A1} with → 0 ← in CASSY Lab 2.

- Start the measurement in CASSY Lab 2 with ⏱.

- Move the pendulum until it reaches the stop. Make sure the pointer for the displacement does not touch the limiting spring. Hold the pendulum!

Note: From the start, the pendulum should always be deflected toward the side with a positive angle of deflection!

- Let the pendulum swing until it comes to rest.

- When the pendulum is at rest again, stop the measurement in CASSY Lab 2 with ⏱.

- Repeat the measurement with greater currents (up to approx. 1.4 A).

b) Studying the transition from underdamping through critical damping to overdamping

Critically damped

- Set the current to approx. 1.5 A.

- Move the pendulum until it reaches the stop and let go. The pendulum should approach the rest position as fast as possible without crossing the rest position.

- If necessary – i.e. if underdamped – slightly increase the current.

- Once the current is set, write it down.

- Repeat the measurement according to the description above.

Note: After recording the curve, switch the power supply off to let the electric magnet for the eddy-current brake cool down.

Overdamped

- Switch the power supply on and set the current to approx. 2 A.

- Repeat the measurement according to the description above.

- After recording the curve, immediately switch the power supply off to avoid overheating the electric magnet for the eddy-current brake.

Measurement Example

Underdamped

Fig. 5 to 7 represent measurement examples for underdamping. The x-axes (time) have different scales.

An alignment according to equation (2) is fit to the data:

$$\varphi(t) = e^{-\delta t} \cdot (A \cdot \cos \omega t + B \cdot \sin \omega t).$$

The following table represents the corresponding currents and fitting parameters.

Table 1: Fitting parameters for underdamping.

I	δ	ω	A	B
A	$\frac{1}{s}$	$\frac{°}{s}$	°	°
0.30	0.13	196.4	138.6	-57.3
0.64	0.50	193.7	138.7	-37.4
1.34	2.10	158.0	130.0	26.7

Critically damped

Fig. 8 shows a measurement example for critical damping.

An alignment according to equation (3) is fit to the data:

$$\varphi(t) = e^{-\delta t} \cdot (A + B \cdot t).$$

In this case, there is one fewer parameter, since the difference $(\delta^2 - \omega_0^2)$ disappears.

Table 2: Fitting parameters for critical damping.

I	δ	A	B
A	$\frac{1}{s}$	°	$\frac{°}{s}$
1.6	4.2	131.5	433.2

Overdamped

Fig. 9 shows a measurement example for overdamping. With so much damping, the pendulum slowly "creeps" from the deflection position to the rest position without crossing the rest position.

An alignment according to equation (4) is fit to the data:

$$\varphi(t) = e^{-\delta t} \cdot \left(A \cdot e^{\lambda t} + B \cdot e^{-\lambda t}\right).$$

δ is no longer the damping coefficient here. In the equation, there are two decreasing exponential functions with exponents $(-(\lambda + \delta) t)$ and $((\lambda - \delta) t)$. The smaller exponent determines the creep.

Table 3: Fitting parameters for overdamping.

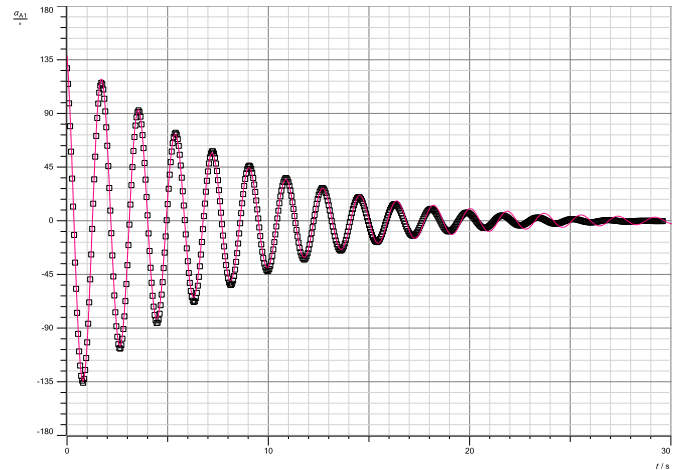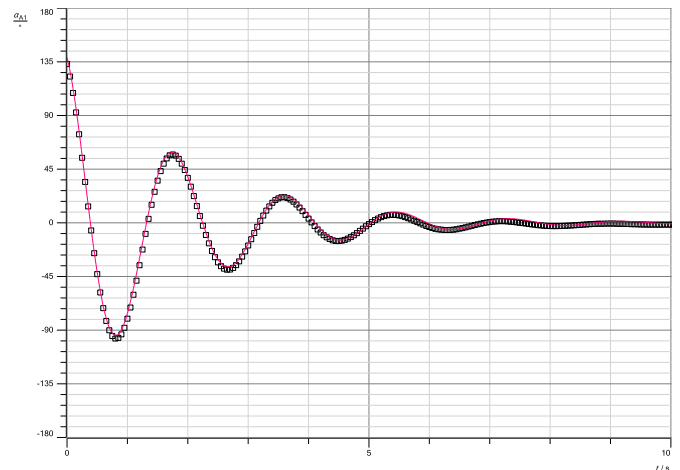| I	δ	λ	A	B
A	$\frac{1}{s}$	$\frac{1}{s}$	°	°
1.9	1.41	0.16	-145.9	283.2

Fig. 5: Underdamped with I = 0.3 A.

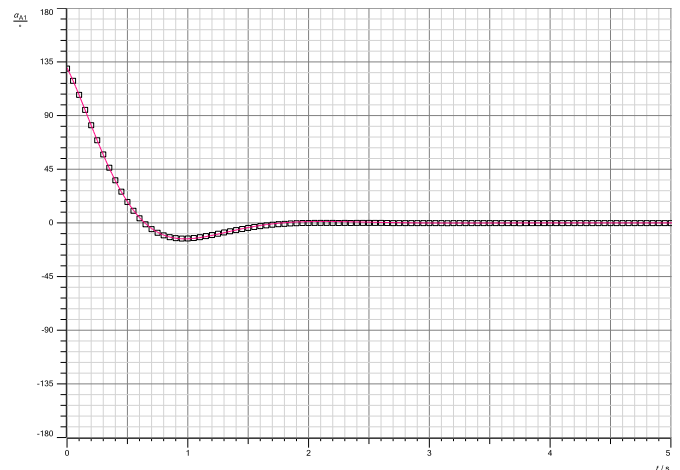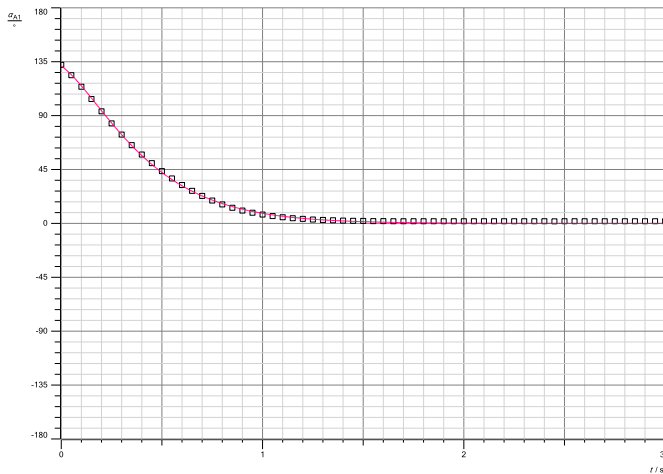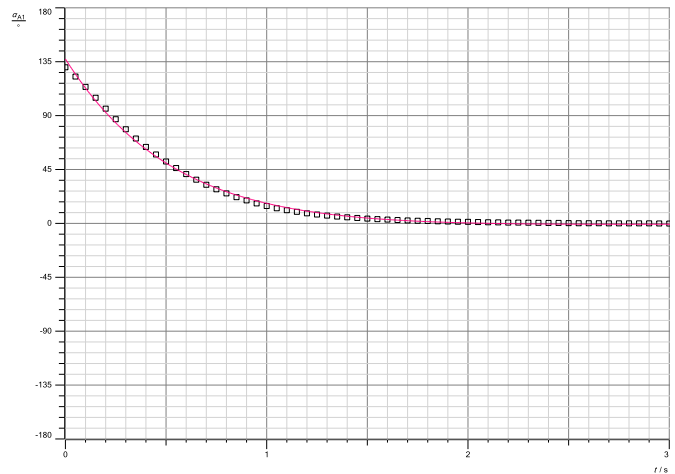Fig. 6: Underdamped with I = 0.64 A.

Fig. 7: Underdamped with I = 1.35 A.

Fig. 8: Critically damped with *I* = 1.6 A.

Fig. 9: Overdamped with *I* = 1.9 A.

Comments

In Tables 1 and 2, you can see the damping coefficient *δ* is a quickly increasing function of the current applied to the electric magnet for the eddy-current brake. The amplitude's exponential decline is a consequence of the damping, which is proportional to the angular velocity. At very slow rotary motions, this damping is in the magnitude of the mechanical friction (very low), and equation (1) gets another term. This effect becomes clear in the small phase shift between the curve fitting and the measurement values in Fig. 5 and Fig. 6 at low amplitudes (large *t* values).

Fig. 8 and 9 have the same scale on the time axis, which allows for direct comparison. Critical damping differs from overdamping in that the pendulum needs less time to come to rest, i.e. until the angle of deflection is 0 again.