Path-time diagrams of rotational motions – time measurements with the counter

Objects of the experiment

- Determination of the angular displacement time diagram
- Determination of the mean angular velocity
- Determination of the mean angular acceleration

Principles

In this experiment, the rotational motion is investigated analogously to the translational motion. Translational motions are described in terms of path s, time t, velocity v and acceleration a. Rotational motions are described in terms of angular displacement φ, time t, angular velocity ω, and angular acceleration α.

For an object rotating about an axis every point on the object has the same angular velocity ω. The angular velocity is the rate of change of angular displacement $\Delta\varphi$. The average angular velocity can be described by the relationship:

$$\overline{\omega} = \frac{\Delta\varphi}{\Delta t} \qquad \text{(I)}$$

The angular acceleration α is the rate of change of angular velocity ω. The average angular acceleration can be described by the relationship:

$$\overline{\alpha} = \frac{\Delta\omega}{\Delta t} \qquad \text{(II)}$$

If the angular acceleration α is constant, in analogy to the translational motion, the following equations represent a complete description of the uniformly accelerated rotational motion:

$$\varphi(t) = \varphi_0 + \omega_0 \cdot t + \frac{1}{2}\alpha \cdot t^2 \quad \text{(angular displacement)} \qquad \text{(III)}$$

$$\omega(t) = \omega_0 + \alpha \cdot t \quad \text{(angular velocity)} \qquad \text{(IV)}$$

φ_0: initial angular displacement

ω_0: initial angular velocity

α: angular acceleration

In this experiment the rotational model is used to investigate the uniform rotational motion and the uniformly accelerated rotational motion.

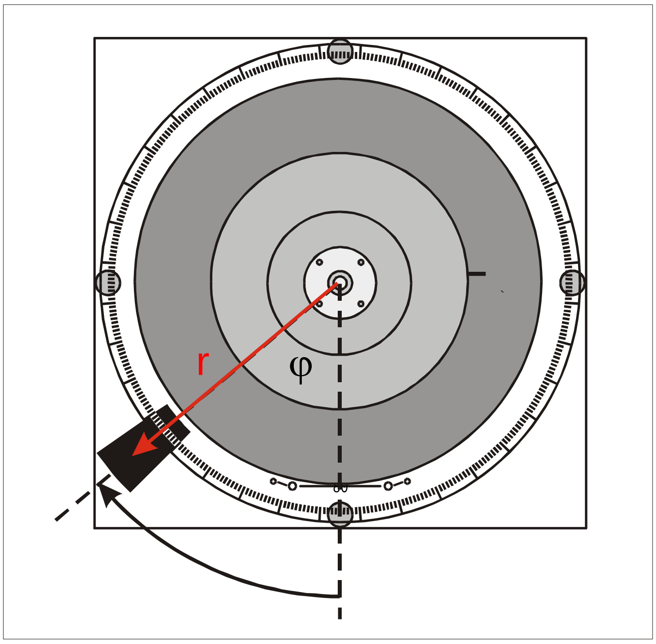

Fig. 1: Rotational motion of a mass (here the black flag) at radius r, from the center of rotation. φ angular displacement.

a) uniform rotational motion

By choosing appropriate conditions for the initial angular displacement (φ_0 = 0) and the angular acceleration (α = 0) in equations (III) and (IV) the following equations result for describing the uniformly rotating flag (Fig. 1).

$$\varphi(t) = \omega_0 \cdot t \qquad \text{(V)}$$

$$\omega(t) = \omega_0 \qquad \text{(VI)}$$

b) uniformly accelerated rotational motion

By choosing appropriate conditions for the initial angular displacement (φ_0 = 0) and the angular velocity (ω_0 = 0) in

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Apparatus

1 Rotational model 347 23
2 Forked light barrier... 337 46
2 Multi-core cable, l = 1.50 m............................... 501 16
1 Counter S 575 471
1 Laboratory stand II, 16 x 13 cm 300 76
1 Simple bench clamp 301 07

equations (III) and (IV) the following equations result for describing the uniformly accelerated rotating flag (Fig. 1).

$$\varphi(t) = \frac{1}{2}\alpha \cdot t^2 \qquad \text{(VII)}$$

$$\omega(t) = \alpha \cdot t \qquad \text{(VIII)}$$

Setup

Set up the rotational model with one flag on the Laboratory stand as shown in Fig. 2. Prepare a transmission thread with a loop at one end. For the uniform rotational motion the length of the thread is determined by the angle α and the stop position, e.g. a chair (Fig. 2). For the uniformly accelerated rotational motion the length of the thread is determined by the condition that during the whole experiment the disk is accelerated.

The accelerating force is generated e.g. by 3 small suspended weights of 1 g each (F = 0.0294 N).

Connect the forked light barrier for the start signal via the multi-core cable to the input E of the counter S. Connect the forked light barrier for the stop signal via the multi-core cable to the input F of the counter S.

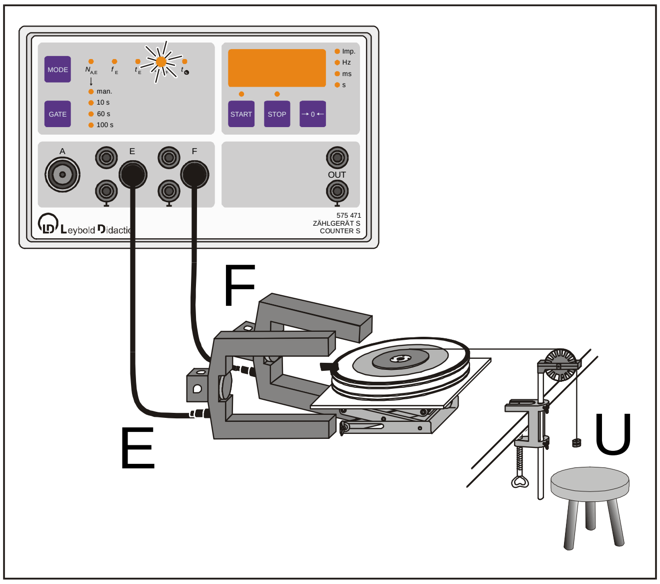


Fig. 2: Experimental setup to determine the angular displacement φ and angular velocity ω as function of time t.
E: light barrier for start signal
F: light barrier for stop signal
U: uniform rotational motion only: stop for weight, e.g. chair

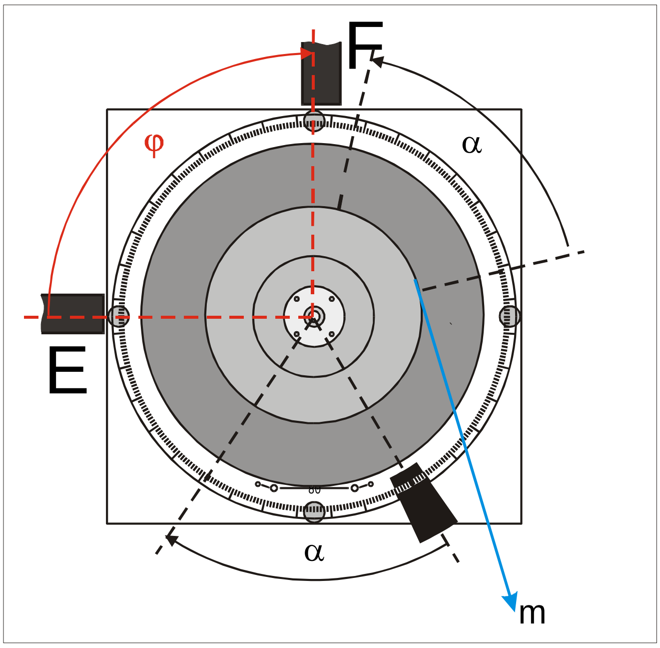


Fig. 3: Schematic sketch for setting up the light barriers for the start signal (E) and stop signal (F) for the uniform rotational motion.
α: angle of winding up the thread (i.e. the angle when the disk is accelerated)
φ: angle between light barriers E and F
m: thread with accelerating weight

Carrying out the experiment

a) Uniform rotational motion

- Place the forked light barrier for the start signal (E) and stop signal (F) at appropriate positions in respect to the rotating flag to measure the time of circulation Fig 3.
- Fix three weights of 1 g to the transmission thread (to the end with no loop).
- Hook up the transmission thread with its loop to the pin of an extra disk with e.g. a radius of r = 5 cm (Fig. 4) and run it over the wheel which is attached by the bench clamp on the edge of the table.
- Wind up the transmission thread by rotating the disk of the rotational model by an angle α , e.g. 80° degree.

Note: There are two possibilities to set the disk into a uniform rotational motion. One method is to stop the weight before the flag enters the light barrier E. Thus the experiment has become reproducible starting conditions.
Alternatively the angle α can be chosen in such a manner that the thread will release itself after rewinding, i.e. at the angle 80° before the rotational flag enters the light barrier E. Due to low friction of the rotational model the disk rotates now uniformly.

- Hold this position of the rotating the disk and select on the counter S the mode button “Measurement of the time delay between inputs E and F ($t_{E\to F}$)” by toggling.
- Set the rotational model into uniform rotational motion by releasing the disk. (The upper disk is accelerated by the falling weight. The counter S measures the time delay automatically when the flag has passed the stop light barrier F).
- Repeat the measurement to obtain the mean average.
- Perform the experiment for several angles φ between the light barrier E and F.

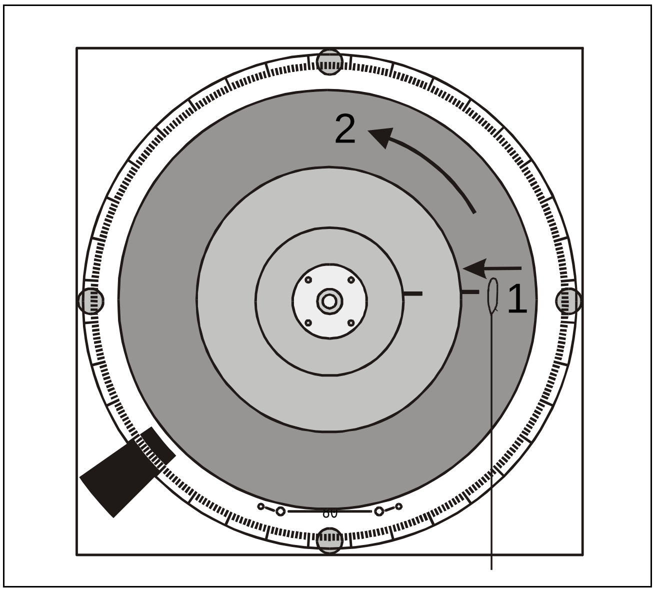
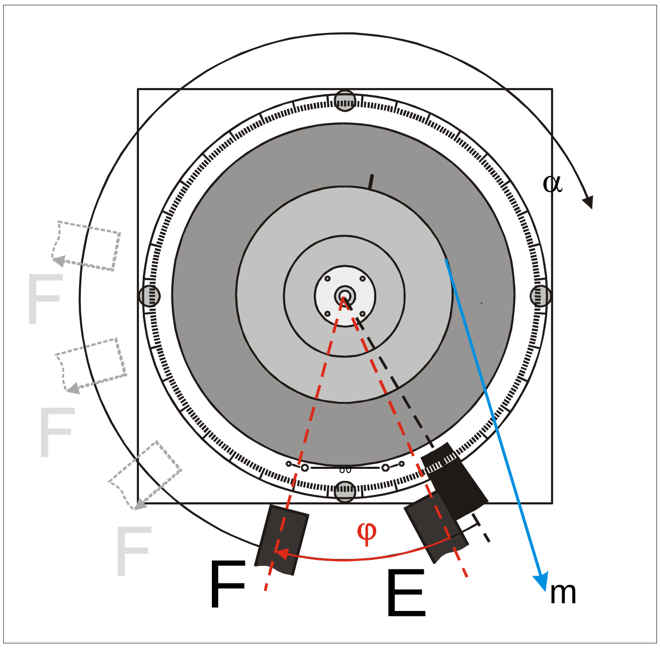


Fig. 4: Schematic sketch for setting the rotational model in uniform or uniformly accelerated rotational motion conveniently:
1. Hook up the transmission thread to a pin by its loop.
2. Wind up the transmission thread by rotating the disk.

Fig. 5: Schematic sketch for setting up the light barriers for the start signal (E) and stop signal (F) for the uniform accelerated rotational motion. The angular change φ of the light barrier F is indicated in grey.
α: angle of winding up the thread (i.e. the angle when the disk is accelerated)
φ: angle between light barriers E and F
m: thread with accelerating weight



b) Uniformly accelerated rotational motion

- Place the forked light barrier for the start signal E in such a manner that the rotating flag will start immediately the measurement upon releasing the upper disk (Fig. 5).
- Place the light barrier for the stop signal F at an angle φ from the light barrier of the start signal E to measure the time delay.
- Fix 3 weights of 1 g to the transmission thread (to the end with no loop) and lay it over the wheel (Fig. 4).
- Hook up the transmission thread with its loop to the pin of an extra disk with e.g. a radius of r = 5 cm.
- Wind up the transmission thread by rotating the disk of the rotational model until the rotating flag is at the position where it would interrupt the light barrier E upon releasing.
- Hold this position of the rotating the disk and select by toggling the mode button on the counter S "Measurement of the time delay between inputs E and F ($t_{E\rightarrow F}$)".
- Release the upper disk of the rotational model. (The disk is accelerated by the falling weight. The counter S measures and displays the time delay automatically when the rotating flag has passed the stop light barrier F).
- Repeat this measurement for each angle φ several times to obtain the mean average.
- Perform the experiment for various angles φ between the light barrier E and F.

Measuring example

a) Uniform rotational motion

Table 1: Delay time (average over 5 measurements) as function of angle φ between the light barriers E and F.

$\frac{\varphi}{\text{deg}}$	$\frac{t}{\text{s}}$
90	0.638
180	1.396
270	1.781
330	2.252

b) Uniformly accelerated rotational motion

Table 2: Time (average over 5 measurements) as function of angular displacement φ.

$\frac{\varphi}{\text{deg}}$	$\frac{t}{\text{s}}$
50	1.407
90	2.166
180	2.977
270	3.570
335	4.051

Evaluation and results

a) Uniform rotational motion

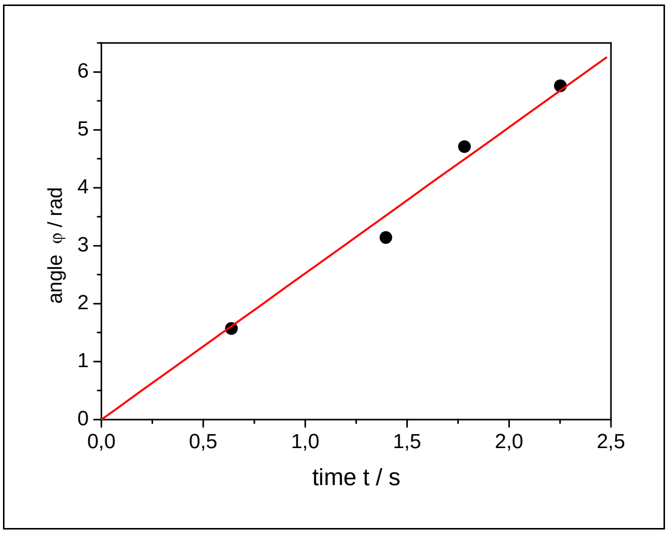


Fig. 6: Angular displacement φ as function of time t. The straight line corresponds to a fit according equation (V).

The φ(t) diagram (i.e. path diagram of the rotating flag) of an uniform rotational motion is a straight line. The slope of the straight line corresponds to the angular velocity ω_0 and can be determined e.g. by fitting a straight line to the data plotted in Fig. 6:

$$\omega = \frac{\Delta\varphi}{\Delta t} = 2.5\,\frac{\text{rad}}{\text{s}}$$

Note: The angular velocity can be determined directly by measuring of the interruption time of the light barrier (see supplementary information).

b) Uniformly accelerated rotational motion

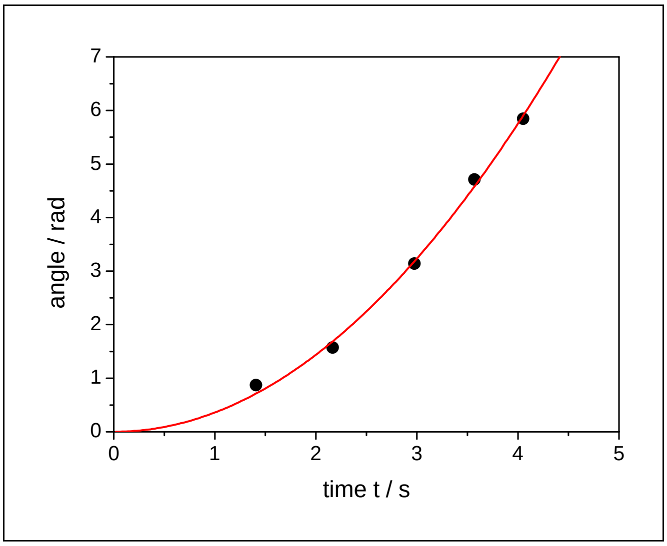


Fig. 7: Angular displacement φ as function of time t. The solid line corresponds to a fit according equation (VII).

The φ(t) diagram (i.e. path diagram of the rotating flag) of an uniformly accelerated rotational motion is a parabola. This can be confirmed by a fit of equation (VII) to the experimental data (Fig. 7).

For determining the angular acceleration (graphically) it is convenient to plot the angle φ as function of t^2 (Fig. 8.). The slope of the straight line corresponds to the angular acceleration α and can be determined either graphically or by fitting a straight line to the data plotted in Fig. 8:

$$\alpha = 0.36\,\frac{\text{rad}}{\text{s}^2}$$

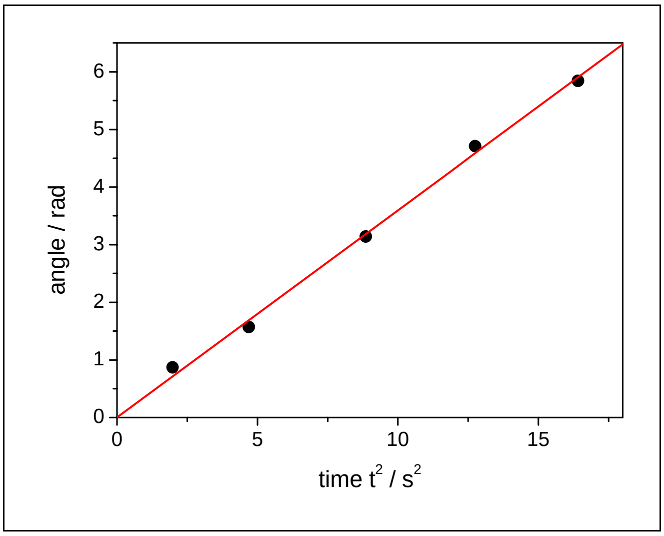


Fig. 8: Angle φ as function of time t^2. The straight line corresponds to a fit of a straight line.

Supplementary information

The average angular velocity of a uniform rotational motion can also be determined directly by measuring of the interruption time t_r when using one light barrier. This experimental procedure is described in the leaflet P1.4.1.1(a).

Further experiments

The experiment can also be used to confirm Newton's equation of motion for the circular motion:

$$M = J \cdot \alpha \qquad \text{(IX)}$$

The different torques M at a constant moment of inertia J are applied by using the different pulley radii (M = r·F = 2.94 mNm, 1.47 mNm, 0.73 mNm). Alternatively, the experiment can be performed with different moments of inertia for a constant torque using extra disks.

When the angular accelerations are plotted as a function of the accelerating torques M, M is found to be proportional to α (with J as the proportionality factor), thus confirming equation (IX).

Alternatively, you can keep the accelerating torque M constant and vary the moment of inertia J. The result here is J proportional to 1/ α (with M as the proportionality factor).