Path-time diagrams of rotational motions – time measurements with the counter

Objects of the experiment

- Determination of the angular displacement time diagram
- Determination of the mean angular velocity
- Determination of the angular acceleration

Principles

In this experiment, the rotational motion is investigated analogously to the translational motion. Translational motions are described in terms of path s, time t, velocity v and acceleration a. Rotational motions are described in terms of angular displacement φ, time t, angular velocity ω, and angular acceleration α.

For an object rotating about an axis, every point on the object has the same angular velocity ω. The angular velocity is the rate of change of angular displacement $\Delta\varphi$. The average angular velocity can be described by the relationship:

$$\overline{\omega} = \frac{\Delta\varphi}{\Delta t} \qquad \text{(I)}$$

The angular acceleration α is the rate of change of angular velocity ω. The average angular acceleration can be described by the relationship:

$$\overline{\alpha} = \frac{\Delta\omega}{\Delta t} \qquad \text{(II)}$$

If the angular acceleration α is constant, in analogy to the translational motion, the following equations represent a complete description of the uniformly accelerated rotational motion:

$$\varphi(t) = \varphi_0 + \omega_0 \cdot t + \frac{1}{2}\alpha \cdot t^2 \quad \text{(angular displacement)} \qquad \text{(III)}$$

$$\omega(t) = \omega_0 + \alpha \cdot t \quad \text{(angular velocity)} \qquad \text{(IV)}$$

φ_0: initial angular displacement

ω_0: initial angular velocity

α: angular acceleration

In this experiment the rotational model is used to investigate the uniform rotational motion and the uniformly accelerated rotational motion.

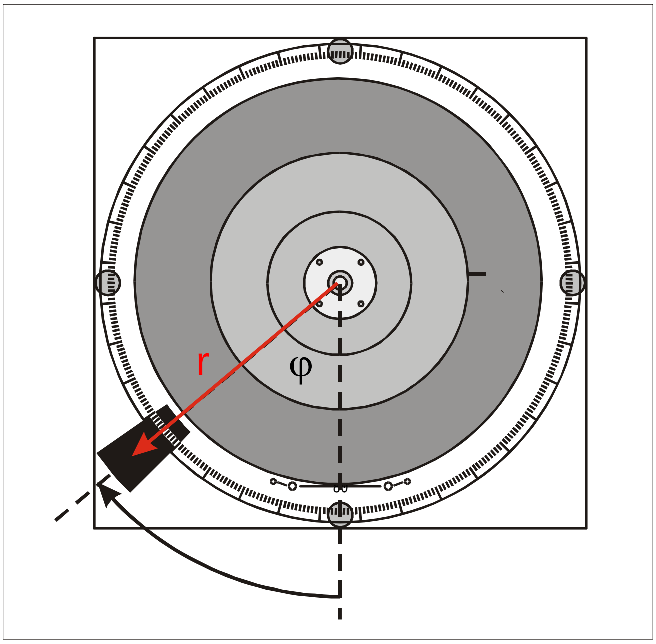

Fig. 1: Rotational motion of a mass (here the black flag) at radius r, from the center of rotation. φ angular displacement.

a) uniform rotational motion

By choosing appropriate conditions for the initial angular displacement (φ_0 = 0) and the angular acceleration (α = 0) in equations (III) and (IV) the following equations result for describing the of the uniformly rotating flag (Fig. 1.)

$$\varphi(t) = \omega_0 \cdot t \qquad \text{(V)}$$

$$\omega(t) = \omega_0 \qquad \text{(VI)}$$

b) uniformly accelerated rotational motion

By choosing appropriate conditions for the initial angular displacement (φ_0 = 0) and the angular velocity (ω_0 = 0) in

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Apparatus

1 Rotational model 347 23
1 Forked light barrier.. 337 46
1 Multi-core cable, l = 1.50 m................................ 501 16
1 Counter S 575 471
1 Laboratory stand II, 16 x 13 cm 300 76
1 Simple bench clamp 301 07

equations (III) and (IV) the following equations result for describing the of the uniformly accelerated rotating flag (Fig. 1.)

$$\varphi(t) = \frac{1}{2}\alpha \cdot t^2 \qquad \text{(VII)}$$

$$\omega(t) = \alpha \cdot t \qquad \text{(VIII)}$$

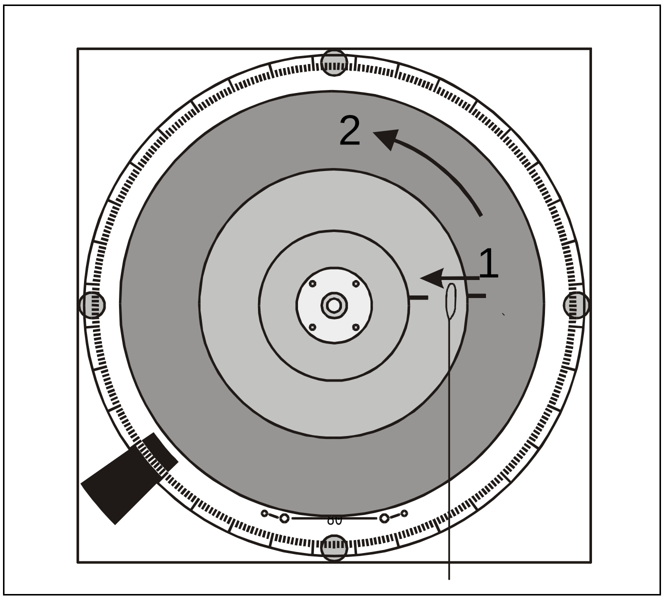

Fig. 3: Schematic sketch for setting the rotational model in uniform or uniformly accelerated rotational motion conveniently:
1. Hook up the transmission thread to a pin by its loop.
2. Wind up the transmission thread by rotating the disk.

Setup

Set up the rotational model with one flag on the Laboratory stand as shown in Fig. 2. Prepare a transmission thread of a length of approximately 100 cm to 150 cm with a loop at one end. Hook up the transmission thread to the pin of an extra disk (e.g. r = 2.5 cm) of the rotation model (Fig. 3) and run it over the wheel which is attached by the bench clamp on the edge of the table.

The accelerating force is generated e.g. by 3 small suspended weights of 1 g each (F = 0.0294 N).

Connect the forked light barrier via the multi-core cable to the input E of the counter S.

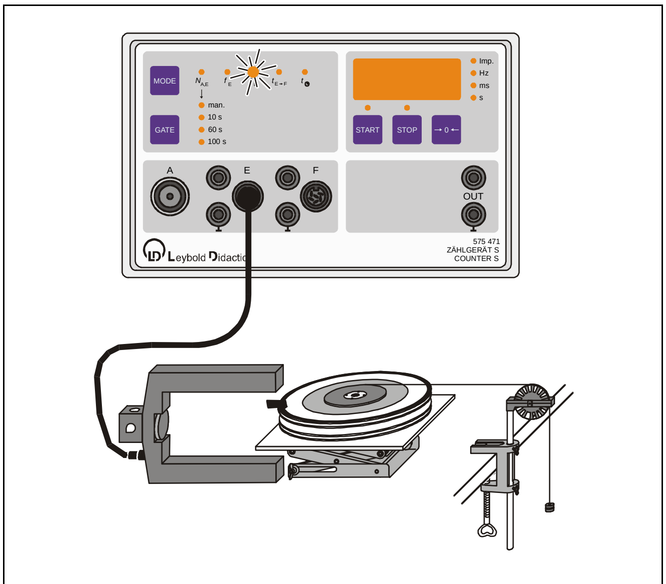

Fig. 2: Experimental setup to determine the angular displacement φ and angular velocity ω as function of time t.

Carrying out the experiment

a) Uniform rotational motion

- Place the forked light barrier on an appropriate position in respect to the rotating flag to measure the time of circulation.
- Fix 3 weights of 1 g to the transmission thread (to the end with no loop) and run it over the wheel.
- Hook up the transmission thread with its loop to the pin of an extra disk with e.g. a radius of r = 5 cm (Fig. 3).
- Wind up the transmission thread by rotating the disk of the rotational model, e.g. one turn to have reproducible starting conditions.
- For the time measurements select “manual time measurement $t_{◷}$” by toggling the mode button on the counter S.
- Set the rotational model into uniform rotational motion by accelerating the upper disks with the falling the weight. The thread will release itself after complete rewinding. Due to low friction of the rotational model the disk rotates now uniformly.
- Measure the time for one round manually (i.e. for an angle φ = 360°) by using the “Start” and the “Stop” button of the counter S.

Hint: The interruption of light emitting diode of the forked light barrier by the rotating flag can be used as Start and Stop aid.

- Repeat the measurement for 2, 3 and 4 turns.

b) Uniformly accelerated rotational motion

- Fix 3 weights of 1 g to the transmission thread (to the end with no loop) and lay it over the wheel.
- Hook up the transmission thread with its loop to the pin of an extra disk with e.g. a radius of r = 2.5 cm.
- Wind up the transmission thread by rotating the disk of the rotational model, e.g. 2 turns.

- Place the forked light barrier on an appropriate position in respect to the rotating flag (i.e. starting position) to measure the time t as function of the angle φ.
- For the time measurements select “manual time measurement $t_{◷}$” by toggling the mode button on the counter S.
- Measure the time for various angles φ manually:

  Push the “Start” button of the counter S when releasing the weight, i.e. when the rotational model starts to rotate.

  Push the “Stop” button of the counter S when the light emitting diode of the forked light barrier is interrupted by the rotating flag of the counter S.

Measuring example

a) Uniform rotational motion

Table 1: Time (average over 5 measurements) as function of angular displacement φ.

$\frac{\varphi}{\text{deg}}$	$\frac{t}{\text{s}}$
360	2,41
720	5,04
1080	7,61
1440	10,80

b) Uniformly accelerated rotational motion

Table 2: Time (average over 5 measurements) as function of angular displacement φ.

$\frac{\varphi}{\text{deg}}$	$\frac{t}{\text{s}}$
30	2,17
60	2,81
90	3,22
120	3,99
150	4,63
180	5,04
210	5,20
240	5,78
270	5,88

Evaluation and results

a) Uniform rotational motion

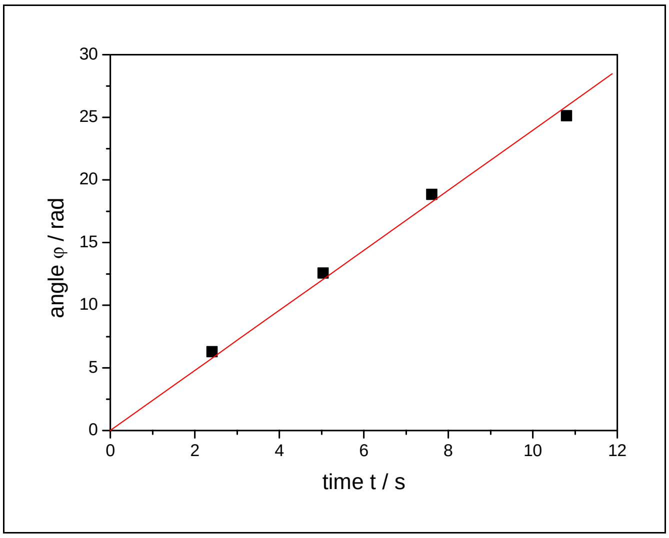

Fig. 4: Angular displacement φ as function of time t. The straight line corresponds to a fit according equation (V).

The φ(t) diagram (i.e. path diagram of the rotating flag) of an uniform rotational motion is a straight line. The slope of the straight line corresponds to the angular velocity ω and can be determined e.g. by fitting a straight line to the data plotted in Fig. 4:

$$\omega = \frac{\Delta\varphi}{\Delta t} = 2.4\,\frac{\text{rad}}{\text{s}}$$

The angular velocity can be determined directly by measuring of the interruption time of the light barrier (see supplementary information).

b) Uniformly accelerated rotational motion

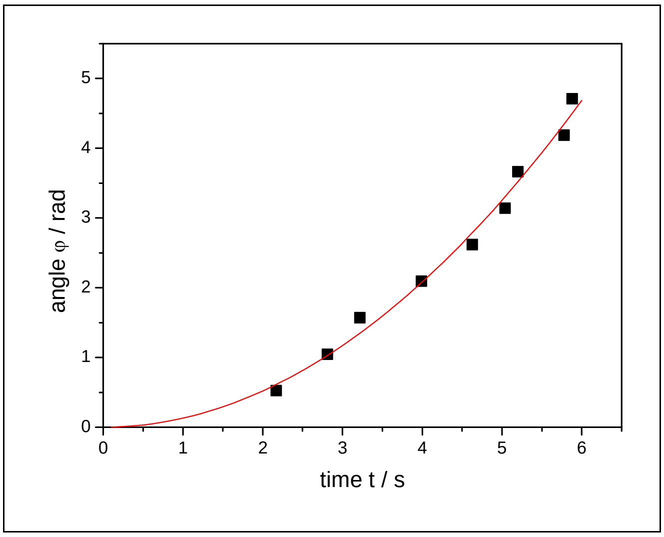

Fig. 5: Angular displacement φ as function of time t. The solid line corresponds to a fit according equation (VII).

The φ(t) diagram (i.e. path diagram of the rotating flag) of an uniformly accelerated rotational motion is a parabola. This can be confirmed by a fit of equation (VII) to the experimental data (Fig. 5).

For determining the angular acceleration (graphically) it is convenient to plot the angle φ as function of t^2 (Fig. 6.). The slope of the straight line corresponds to the angular acceleration α and can be determined either graphically or by fitting a straight line to the data plotted in Fig. 6:

$$\alpha = 0.13 \frac{\text{rad}}{\text{s}^2}$$

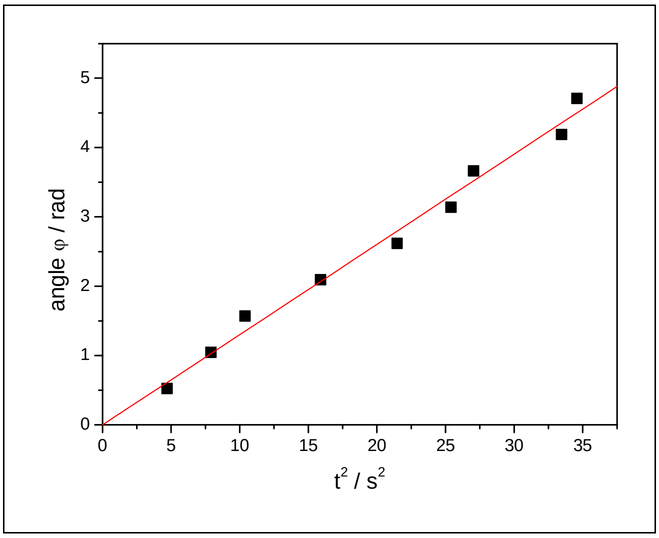

Fig. 6: Angle φ as function of time t^2. The straight line corresponds to a fit of a straight line.

Supplementary information

The average angular velocity of a uniform rotational motion can be determined directly by measuring of the interruption time t_r of the light barrier:

- For the time measurements select "time measurement at input E", i.e. t_E by toggling the mode button on the counter S.
- Set the rotational model into uniform rotational motion as described in part a) of section 'Carrying out the experiment'.
- Measure the interruption time for 1, 2, 3 and 4 turns.

Table 3: Interruption time t_r and angular velocity ω as function of angular displacement φ.

$\frac{\varphi}{\text{deg}}$	$\frac{t_r}{\text{s}}$	$\frac{\omega}{\text{rad}\cdot\text{s}^{-1}}$
360	0.0669	2.61
720	0.0712	2.45
1080	0.0752	2.32
1440	0.0801	2.18

Due to the reproducible initial conditions, i.e. the acceleration of the rotational model to a constant initial velocity ω_0, the result of Table 1 and Table 3 can be summarized in the following plot (compare equation (VI)):

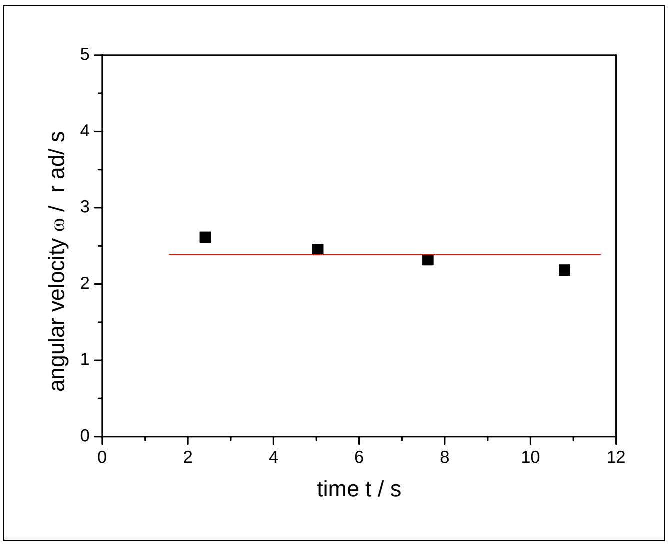

Fig. 4: Angular velocity ω determined from the interruption time t_r as function of time t. The straight line corresponds to the mean average of ω.

The mean average is determined to

$$\overline{\omega} = \frac{\Delta\varphi}{\Delta t} = 2.4 \frac{\text{rad}}{\text{s}}$$

and is thus in agreement with the value found form Fig. 4. The slight decrease of the angular velocity with increasing time is due to the inevitable friction of the rotational model.

Further experiments

The experiment can also be used to confirm Newton's equation of motion for the circular motion:

$$M = J \cdot \alpha \qquad \text{(IX)}$$

The different torques M at a constant moment of inertia J are realized using the different pulley radii (M = r·F = 2.94 mNm, 1.47 mNm, 0.73 mNm). Alternatively, you can realize different moments of inertia for a constant torque using extra disks.

When the angular accelerations are plotted as a function of the accelerating torques M, M is found to be proportional to α (with J as the proportionality factor), thus confirming equation (IX).

Alternatively, you can keep the accelerating torque M constant and vary the moment of inertia J. The result here is J proportional to 1/ α (with M as the proportionality factor).