Determining the density of liquids using the plumb bob

Objects of the Experiments

- Determine the density of pure water
- Determine the density of ethanol solutions in water as a function of the ethanol concentration by volume

Principles

Regardless of aggregate state, the following is true of the density of substances ρ:

$$\rho = \frac{m}{V}$$

where m = Mass
V = Volume

In order to find the density of liquids, it is possible to determine the buoyancy updraught on a body of known volume. When immersed in the liquid, the body experiences an updraught and therefore appears to weigh less. The magnitude of the updraught is equal to the weight of the displaced liquid and therefore proportional to the density of the liquid ρ_{Fl}:

$$F_A = \rho_{Fl} \cdot V \cdot g$$

where
F_A = updraught
g = acceleration due to gravity

The apparent reduction in weight F is compensated for by adding weights of mass m to the pan of the hydrostatic balance:

$$F = m \cdot g\ .$$

Based on the mass m, as determined by this means, and the already known volume V of the body being used for the experiment, it is possible to calculate the density of the liquid.

This experiment investigates a mixture or solution of two liquids of differing densities ρ_1 and ρ_2. The concentration by volume c is as follows:

$$c = \frac{V_1}{V_1 + V_2}$$

The following is true of the component masses of the liquids V_1 and V_2:

$$m_1 = \rho_1 \cdot V_1 \text{ and } m_2 = \rho_2 \cdot V_2$$

Therefore, the mass of the whole mixture is as follows:

$$m = m_1 + m_2 = \rho_1 \cdot V_1 + \rho_2 \cdot V_2$$

Consequently the density of the mixture is

$$\rho = \frac{m}{V} = \frac{m_1 + m_2}{V_1 + V_2}$$

or

$$\rho = \frac{\rho_1 \cdot V_1 + \rho_2 \cdot V_2}{V_1 + V_2}$$

or

$$\rho = \rho_1 \cdot \frac{V_1}{V_1 + V_2} + \rho_2 \cdot \frac{V_2}{V_1 + V_2}$$

That is in terms of the concentration c:

$$\rho = \rho_1 \cdot c + \rho_2 \cdot (1 - c)$$

or

$$\rho = (\rho_1 - \rho_2) \cdot c + \rho_2$$

In this experiment, solutions are prepared using ethanol and water.

Kem 08-13

Apparatus

1 Hydrostatic balance 315 011
1 Set of weights, 10 mg to 200 g 315 31
1 Plumb bob 362 025

2 Measuring cylinders, 100 ml, ... 665 754
1 Stirring thermometer, -30...+110 °C/1 K 382 21
1 Ethanol, denaturated, 1 l................................. 671 9720

Setup

Note:
Set up the hydrostatic balance somewhere where it is least likely to experience any shaking, radiated heat or draughts.

Fig.1: Hydrostatic balance with one short arm and one long arm.

- Set up the hydrostatic balance with one long arm and one short arm and put the pans on it.
- Move the balance beam up far enough for the plumb bob can be suspended underneath the pan on the shorter arm.
- Carefully suspend the plumb bob dry from that pan and place weights adding up to 30 g (20 g + 10 g) on the other pan.
- Align the hydrostatic balance with the help of its calibration nuts such that the needle points to the centre of the scale.

Carrying out the experiment

Note:
Failing to fully immerse the body in the liquid being used for the measurement, any friction between the bob and the sides of the measuring cylinder, air bubbles associated with the plumb bob and droplets of liquid on the line from which the bob is suspended can all cause the results of the measurement to be erroneous.

a) Measurement using water

- Fill a measuring cylinder with approximately 90 ml of (distilled) water.
- Measure the temperature of the water.

  Place the measuring cylinder under the short arm of the hydrostatic balance.
- Carefully put the plumb bob into the cylinder, moving the latter so that the bob is freely suspended inside (without touching the cylinder walls).
- Carefully lower the bob until it is fully submerged.
- Add weights to the pan on the short balance arm until the needle points back to the centre. Adjust the height of the balance beam if necessary.

b) Measurements using solutions of ethanol in water:

- First make a measurement with pure ethanol and then with aqueous ethanol solutions.
- In order to prepare an aqueous ethanol with the desired concentration by volume c, proceed as follows:
  Fill a measuring cylinder with ethanol of the following volume:

  $$V_1 = c \cdot 100 \text{ ml}$$

  Fill a second measuring cylinder with water of the following volume:

  $$V_2 = (1 - c) \cdot 100 \text{ ml}$$

  Carefully mix the two liquids together in one of the cylinders.
  Then fill the other cylinder with about 90 ml of the solution.
- You should always carefully clean the plumb bob and the measuring cylinders when doing this experiment.

Note: The amount of ethanol used up will be very much reduced if the concentrations in a given set of measurements are suitably halved, e.g. by mixing 50 ml of a given solution with 50 ml of water.
The following dilutions can be made:
$c = 1 \rightarrow c = 0.5;$
$c = 0.8 \rightarrow c = 0.4 \rightarrow c = 0.2 \rightarrow c = 0.1$
$c = 0.6 \rightarrow c = 0.3$

Measuring example

a) Measurement using water

Temperature: T = 20°C

Mass of weights on pan: m = 9.98 g

b) Measurements using solutions of ethanol in water:

Tab. 1: Measurement results

	Concentration	Mass of weights on pan
	c	$\frac{m}{g}$
Water	0	9.98
	0.1	9.84
	0.2	9.72
	0.3	9.60
	0.4	9.43
	0.5	9.24
	0.6	9.02
	0.7	8.79
	0.8	8.52
	0.9	8.24
Ethanol	1	7.89

Evaluation:

The volume of the plumb bob V = 10 cm³.

a) Water

The density of water is therefore given by the following:

$$\rho_{\text{Water}} = \frac{m}{V} = 0.998\,\frac{\text{g}}{\text{cm}^3}$$

Tab. 2: Literature values for water.

Temperature	Density
$\frac{T}{°C}$	$\frac{\rho}{\frac{g}{cm^3}}$
16	0.9989
18	0.9986
20	0.9982
22	0.9978
24	0.9973
26	0.9968
28	0.9962
30	0.9956

This means the results of the measurements are well in agreement with quoted values taking into account the attainable accuracy (10 mg).

b) Aqueous ethanol solutions

The density of ethanol is also well in keeping with book values, as follows:

$$\rho_{\text{Ethanol}} = 0.789\,\frac{\text{g}}{\text{cm}^3}$$

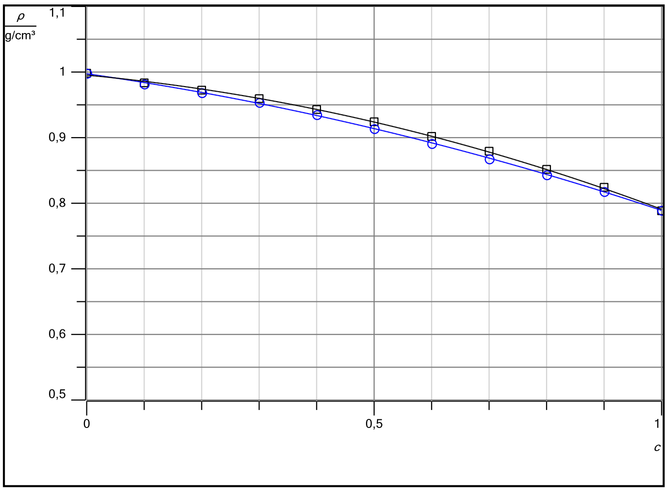

Fig. 2: Density as a function of concentration
(Squares: calculated values, circles: book values for T = 20°C).

Notes:
The contraction in volume when a solution is prepared is not taken into account when determining the concentration. The volume of the solution is not therefore equal to the volumes of the two components. For a concentration c=0.5, for example, the calculated result for the average of the two densities is as follows:

$$\rho_{0.5} = \frac{\rho_1 + \rho_2}{2} = 0.894\,\frac{g}{cm^3}.$$

The measured value

$$\rho_{0.5} = 0.924\,\frac{g}{cm^3}$$

is larger, since the actual volume of the solution is smaller. Ethanol solvents commonly already contain a small quantity of water.

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