Mechanics

Acoustics

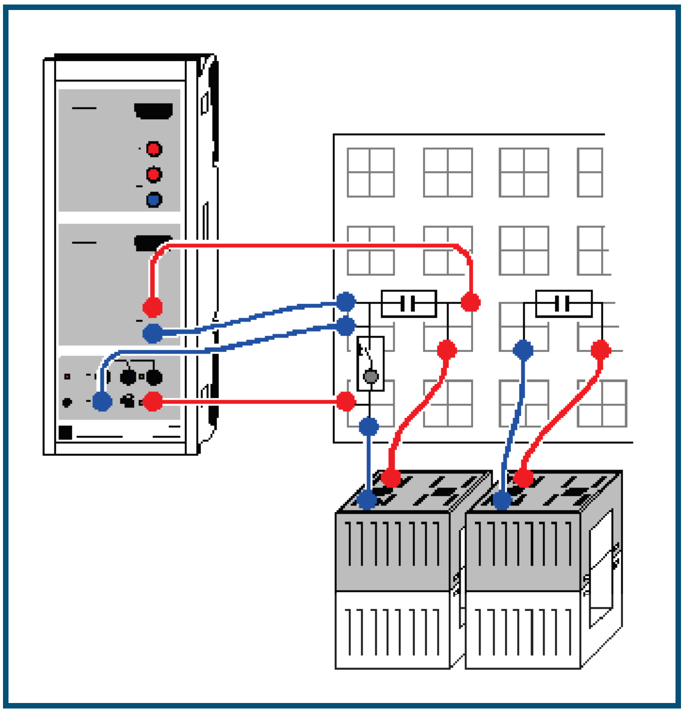
Fourier analysis

Fourier analysis of an electric oscillator circuit

Description from CASSY Lab 2

For loading examples and settings, please use the CASSY Lab 2 help.

Coupled oscillators



Experiment description

The frequency spectrum of coupled electric oscillator circuits is compared with the spectrum of an uncoupled oscillator circuit. The Fourier-transformed signal of the coupled oscillator circuits shows the split into two distributions lying symmetrically around the uncoupled signal, with their spacing depending on the coupling of the oscillator circuits.

Equipment list

1	Sensor-CASSY	524 010 or 524 013
1	CASSY Lab 2	524 220
1	Plug-in board DIN A4	576 74
1	STE key switch	579 10
2	STE capacitors 1 µF, 5 %	578 15
2	Coils with 500 turns	562 14
4	Pairs of cables, 50 cm, red and blue	501 45
1	PC with Windows XP/Vista/7	

Experiment setup (see drawing)

Assemble the first oscillator circuit as shown in the drawing. The capacitor voltage is measured at input B of Sensor-CASSY. At the start of the experiment the capacitor is charged from voltage source S. The oscillation is started with the key switch, which shorts the voltage source S.

Assemble the second oscillator circuit separately. To couple the oscillators, place its coil directly beside the first coil.

Remark

You can also use relay R instead of the key switch. However, this can bounce so hard when switching that the oscillation is disturbed in the first few milliseconds.

Carrying out the experiment

- Load settings
- Set the charging voltage U_{B1} at the capacitor to about 9.5 V using the knob at voltage source S.
- Start the measurement with ⏱ (software waits for the trigger signal).
- Close the oscillator circuit with the key switch (generates trigger signal).
- Place the coil of the second oscillator directly beside that of the first to couple the oscillators.
- Start the measurement with ⏱ (software waits for the trigger signal).
- Close the oscillator circuit with the key switch (generates trigger signal).

Evaluation

The uncoupled case produces a damped harmonic oscillation. The coupled oscillation is a beat with the same envelope and the same oscillation frequency.

In the uncoupled case, the **Frequency Spectrum** display shows only one peak, for which the frequency can be calculated as the peak center.

In the coupled case the frequency splits symmetrically into two frequencies. The amplitudes are just half as great as in the uncoupled case, and the interval depends on the coupling.

Given the differential equations of the coupled oscillation circuits

$$L\ddot{I}_1 + kL\ddot{I}_2 + I_1 / C = 0$$

$$L\ddot{I}_2 + kL\ddot{I}_1 + I_2 / C = 0$$

with coupling k ($0 \le k < 1$), we can calculate the two natural frequencies ω_1 and ω_2 as

$$\frac{\omega_0}{\sqrt{1+k}} = \omega_1 < \omega_0 < \omega_2 = \frac{\omega_0}{\sqrt{1-k}}.$$

In particular, the oscillation frequency of the coupled system is equal to

$$\frac{\omega_1 + \omega_2}{2} = \frac{\omega_0}{\sqrt{1-k^2}} \approx \omega_0$$

and is thus practically unchanged with respect to the uncoupled system (for small k values).

Remark

The split into two peaks of precisely the same size is only possible with completely identical oscillators. This is not always precisely the case due to the tolerances of the inductances L and capacitances C.