

## Two-beam interference of water waves

### Objects of the experiment

- Investigating interference through double excitation of circular water waves as a function of the spacing between the two exciters and the wavelength.
- Investigating the interference of water waves behind a double slit.
- Comparison of interference patterns.

### Principles

“Interference” is the superposing of coherent circular waves at their point of intersection. At some points, the waves are amplified, at others attenuated or even canceled out entirely.

The interference phenomenon at a particular location depends on the shift of the interfering circular waves with respect to each other, or to put it another way, the “path difference” of the two waves at this location.

For a path difference

$$\Delta s = n \cdot \lambda \quad \text{where } n = 0, \pm 1, \pm 2, \dots \quad (\text{I}),$$

the oscillations of the individual waves are added together; in other words, maximum amplification occurs.

For a path difference

$$\Delta s = \left(n + \frac{1}{2}\right) \cdot \lambda \quad \text{where } n = 0, \pm 1, \pm 2, \dots \quad (\text{II}),$$

The amplitudes are subtracted, i.e. the two waves cancel each other out – assuming that they have the same amplitude.

The points of equal path differences lie on hyperbolas (see Fig. 2) for which the focal points are the exciter centers. Their location can be described by the angle  $\alpha$  (see Fig. 3) which they form with the center axis between the exciter centers.

For the maxima, we can say

$$\sin \alpha = n \cdot \frac{\lambda}{d} \quad \text{where } n = 0, \pm 1, \pm 2, \dots \quad (\text{III}),$$

and for the minima

$$\sin \alpha = \left(n + \frac{1}{2}\right) \cdot \frac{\lambda}{d} \quad \text{where } n = 0, \pm 1, \pm 2, \dots \quad (\text{IV})$$

( $d$ : exciter distance,  $\lambda$ : wavelength)

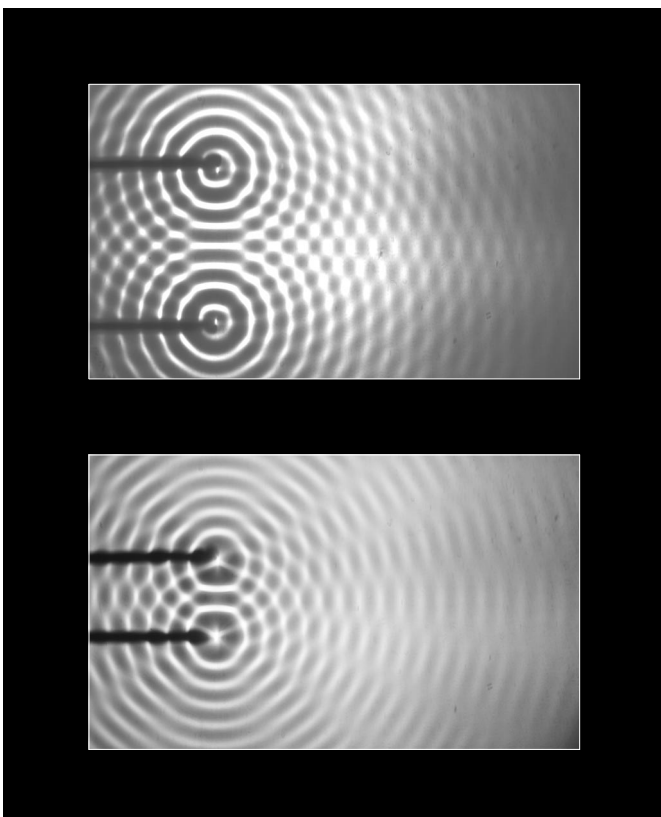


Fig. 1 Interference of two coherent circular waves (photographs)  
Top: distance between point-type exciters = 8 cm  
Bottom: distance between point-type exciters = 4.2 cm

**Apparatus**

1 Wave trough with motor stroboscope . . . 401 501

additionally required:

Dish soap,  
transparencies, transparency pens, adhesive tape,  
ruler, protractor

In the wave trough, coherent circular waves are generated, e.g. using two point-type exciters, which are jointly connected to the membrane of the supply unit to generate circular waves of the same frequency and amplitude. The reflection of circular water waves at a straight obstacle produces the same result. The “mirror image” of the exciter center forms the second exciter center.

When straight wave fronts are incident on an obstacle with two narrow slits, coherent circular waves are also formed behind the slits. Double excitation and double slits generate identical interference phenomena.

**Setup**

Set up the experiment as shown in Fig. 4.

- Set up the wave trough so that it is not subject to shocks and vibrations; observe all information given in the Instruction Sheet.
- Connect two point-type exciters for double excitation 8 cm apart as shown in Fig. 5.
- Place the exciter for straight waves, the obstacle with four slits and the cover slides so that they are within easy reach when needed.
- Attach a transparency to the observation screen (g) using adhesive tape.

**Carrying out the experiment**

**a) Two-beam interference through double excitation:**

- If necessary, rotate the stroboscope disk out of the beam path using knurled screw (f) so that the glass pane in the bottom of the wave trough is completely illuminated.
- Using knob (e), set a frequency of approx. 25 Hz, and carefully increase the excitation amplitude using knob (d) until wave fronts are clearly visible (see Instruction Sheet for wave trough).
- Vary the immersion depth as necessary with adjusting screw (h1).
- Observe the position and number of interference maxima and minima.
- Sketch the excitation centers and the interference hyperbolas on the transparency.
- Measure the wavelength  $\lambda$ , the exciter spacing  $d$  and the directions  $\alpha$  at which the interference minima appear. Be sure to take the image scale into consideration to determine the actual wavelength (see Instruction Sheet for wave trough).
- Reduce the distance between exciters to 4.2 cm and repeat the experiment steps.
- Compare the two interference patterns.
- Set frequencies from approx. 10-40 Hz in steps of 5 Hz. For each frequency, note the wavelength, the position and the number of interference hyperbolas. Draw some additional interference patterns on other transparencies to permit a quantitative evaluation. Compare the interference patterns.

*Note: the change in the position of the interference hyperbolas can be observed particularly clearly by continuously decreasing or increasing the frequency.*

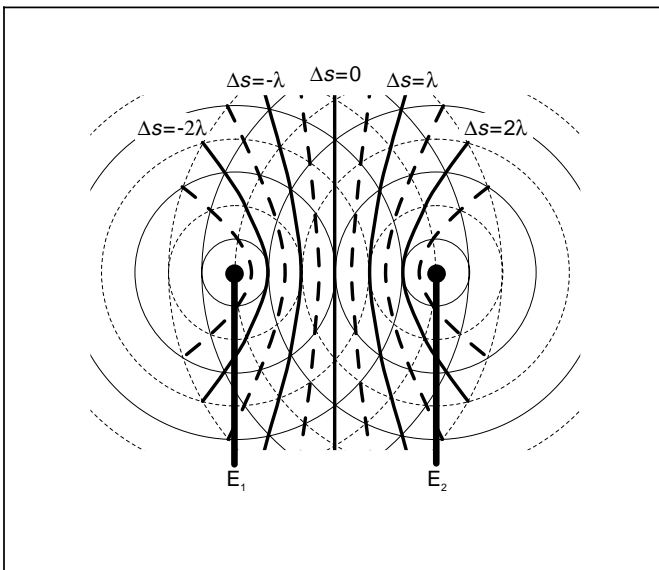
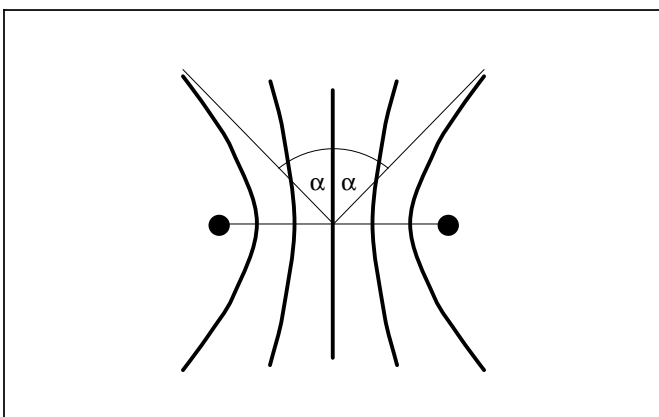


Fig. 2 Schematic representation of interference of two coherent circular waves  
E<sub>1</sub>, E<sub>2</sub> point-type exciters

Fig. 3 Direction  $\alpha$  of interference hyperbolas



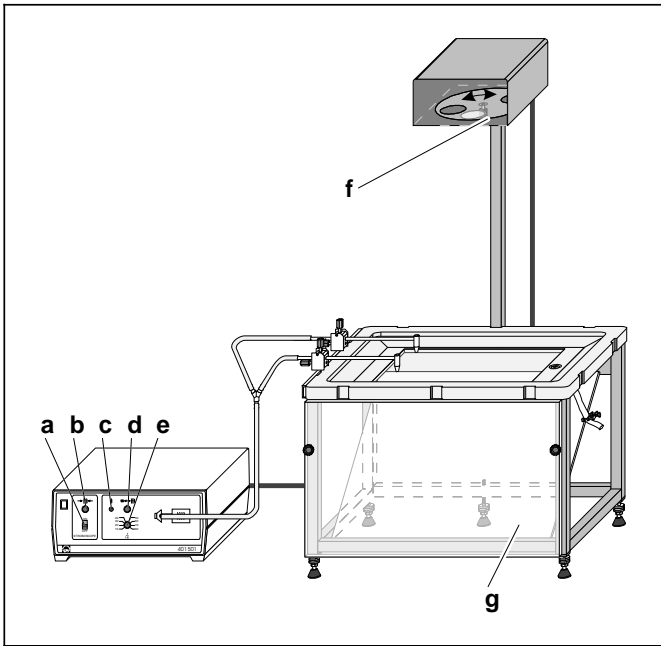


Fig. 4 Experiment setup for two-beam interference  
 a Stroboscope switch  
 b Knob (for fine adjustment of stroboscope frequency)  
 c Pushbutton (single-wave excitation)  
 d Knob (for adjusting amplitude of wave excitation)  
 e Knob (for adjusting frequency of wave excitation)  
 f Knurled screw (for manually turning stroboscope disk)  
 g Observation screen

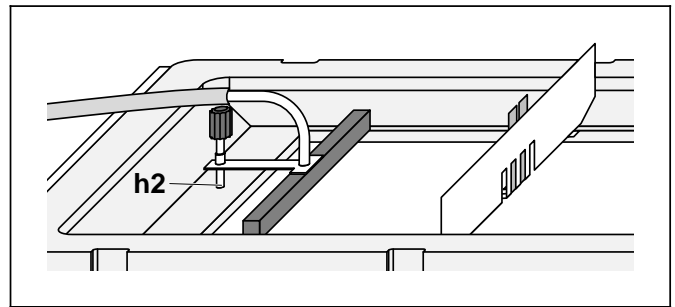
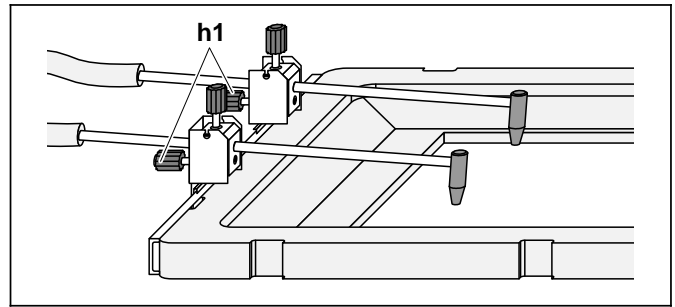


Fig. 5 Connecting the double exciter  
 h1 Adjusting screws (for setting immersion depth)

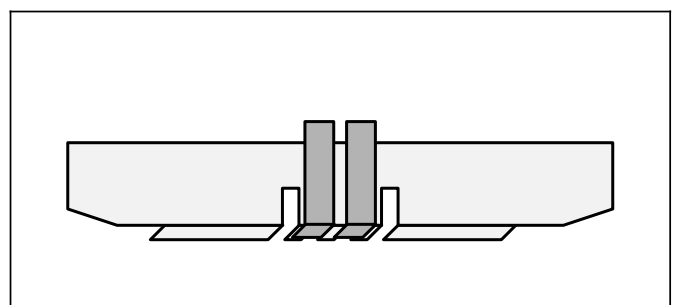
Fig. 6 Connecting the exciter for straight waves and experiment setup for two-beam interference behind a double slit  
 h2 Adjusting screw (for setting immersion depth)

**b) Two-beam interference at a double slit:**

- Remove the point-type exciter and place the obstacle with four slits in the center of the wave trough precisely under the lamp.
- Connect the exciter for straight waves as shown in Fig. 6 and set it up parallel to the obstacle at a distance of 5 cm.
- Cover the two inner slits using the narrow cover slides as shown in Fig. 7 (distance between slit midpoints: 4.2 cm)
- Set a frequency of approx. 25 Hz, and carefully increase the excitation amplitude until wave fronts are clearly visible (see Instruction Sheet for wave trough).
- Vary the immersion depth as necessary with adjusting screw (h2) .
- Observe the position and number of interference maxima and minima.
- Sketch the locations of the slits and the interference hyperbolas on a transparency.
- Measure the wavelength  $\lambda$ , the exciter spacing  $d$  and the directions  $\alpha$  at which the interference minima appear.
- Compare the interference pattern with that of the double exciter (spacing 4.2 cm,  $f = 25$  Hz).
- Change the distance  $d$  between slit midpoints by covering up first the second and fourth slits and then the two outer slits as shown in Fig. 7 and repeat the experiment steps for each combination.

- Compare the interference patterns of the two interference sketches.
- Set frequencies from approx. 10-40 Hz in steps of 5 Hz. For each frequency, note the wavelength, the position and the number of interference hyperbolas. Draw some additional interference patterns on other transparencies to permit a quantitative evaluation. Compare the interference patterns.

Fig. 7 Double slit with 4.2 cm spacing between slit midpoints



**Measuring example**

Two photographs with measuring examples for two-beam interference through double excitation are shown in Fig. 1.

Table 1: Directions  $\alpha$  of the first through third interference hyperbolas for interference minima at  $\lambda = 1.1$  cm and different exciter distances  $d$

$\frac{d}{\text{cm}}$	experiment			equation (III)		
	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_1$	$\alpha_2$	$\alpha_3$
8.0	5°	12°	22°	3.9°	11.9°	20.1°
4.2	7°	22°	39°	7.5°	23.1°	40.9°

Fig. 8 shows two photographs with measuring examples for interference through double excitation and interference behind a double slit.

**Results**

**a) Two-beam interference with the double exciter:**

The circular waves generated by the double exciter are superposed at the points where they meet. The areas with no wave motion indicate cancellation (minima).

The minima and maxima are positioned along hyperbolas with the excitation centers as focal points. The relationships given in the section "Principles" for the positions of the hyperbolas are confirmed experimentally.

The interference patterns are determined by the distance between exciters, and by the wavelength. The number of hyperbolas increases with the exciter spacing and the wavelength, and the hyperbolas open further.

**b) Two-beam interference at a double slit:**

The interference patterns occurring behind the double slit closely resemble those of the double exciter. According to

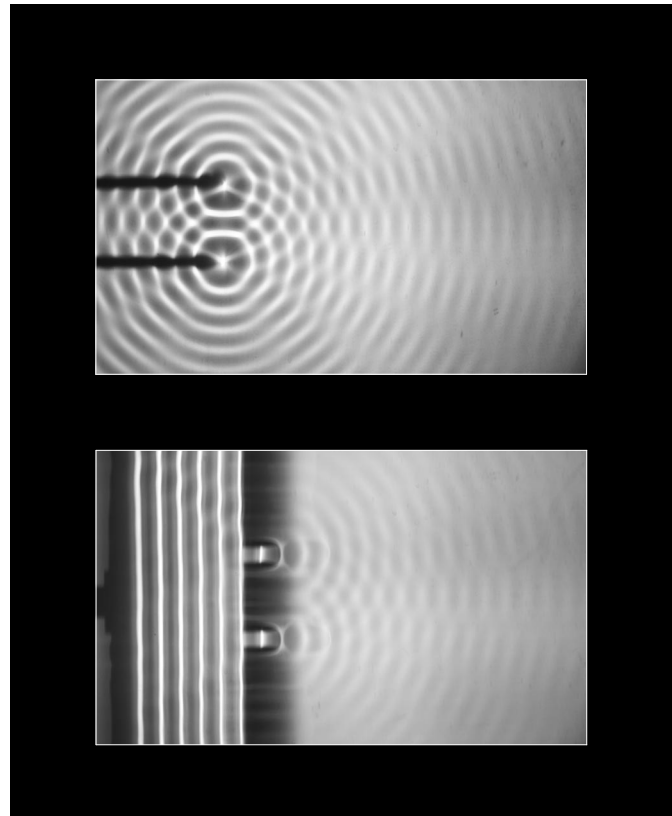


Fig. 8 Two-beam interference of water waves (photographs)  
 Top: double excitation  
 Bottom: diffraction at a double slit.

Huygens' principle, two new circular waves with identical frequency and amplitude (coherent circular waves) are generated at both slits at the same time. The interference structure is identical to that of the double exciter.

The interference patterns are determined by the distance between slit midpoints, and by the wavelength. The number of hyperbolas increases with the distance between slit midpoints and with the wavelength, and the hyperbolas open further.