

## Standing longitudinal waves on an helical spring

### Objects of the experiment

- Investigating longitudinal waves in helical springs fixed at both ends.
- Generating standing waves as a function of the excitation frequency  $f$ .
- Determining the wave velocity  $v$ .
- Investigating the effect of the spring tension on the phase velocity  $v$  and the wavelength  $\lambda$ .

### Principles

A wave is formed when two systems capable of coupled oscillation sequentially execute oscillations of the same type. One example of this is the propagation of a longitudinal wave on a helical spring. The propagation velocity  $v$  of the oscillation

state is related to the oscillation frequency  $f$  and the wavelength  $\lambda$  through the formula

$$v = \lambda \cdot f \quad (I)$$

This is termed the wave velocity or phase velocity. We can say

$$v = \sqrt{\frac{D}{m_0}} \cdot s \quad (II)$$

$D$ : spring constant,  $m_0$ : mass of spring,  $s$ : length of spring

When the helical spring is fixed at both ends and excited to oscillation, reflections occur at both ends, and the outward and reflected waves are superposed. Standing waves form at certain excitation frequencies as stationary oscillation patterns. The distance between two oscillation nodes or two antinodes of a standing wave corresponds to one half the wavelength. For a standing wave with  $n$  oscillation antinodes, we can say

$$\frac{\lambda_n}{2} = \frac{s}{n} \quad \text{with } n = 1, 2, 3, \dots \quad (III)$$

For the excitation frequencies, (I) and (II) give us

$$f_n = v \cdot \frac{n}{2s} \quad (IV)$$

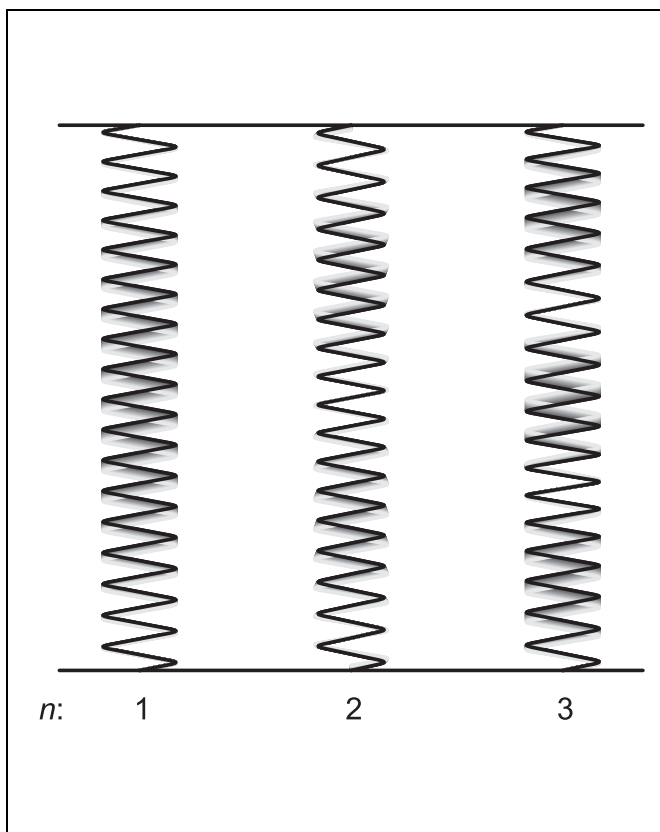
respectively

$$f_n = \sqrt{\frac{D}{m_0}} \cdot \frac{n}{2} \quad (V)$$

Thus, a change in the length  $s$  of the extended helical spring does not change the respective frequency  $f_n$  required to excite  $n$  oscillation antinodes.

In this experiment, two different helical springs are mounted vertically one after another and the bottom end is caused to oscillate by means of an electric motor with an oscillation lever. The excitation frequency is continuously adjustable using a function generator. The length  $s$  of the extended spring can be varied easily by adjusting the suspension material.

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**Apparatus**

1 Helical spring, 5 N; 0.10 N/cm . . . . .	352 7
1 Helical spring, 5 N; 0.25 N/cm . . . . .	352 8
1 Rubber string . . . . .	200 66 629
1 STE motor with flywheel . . . . .	562 791
1 Function generator S12, 0.1 Hz – 20 kHz	522 62
1 Transformer, 6 V AC, 12 V AC/30 VA . . . .	562 73
1 Pair of cables, 1 m, red and blue . . . . .	501 46
2 Stand bases MF . . . . .	301 21
1 Clamping block MF . . . . .	301 25
1 Stand rod, 25 cm . . . . .	301 26
1 Stand rod, 50 cm . . . . .	301 27
1 Pair of pointers . . . . .	301 29
1 Support clips, for plugging in . . . . .	314 04
1 Tape measure, 1.5 m/1 mm . . . . .	311 78

- Hook the end of helical spring 1 into the eyelet of the oscillation lever and attach the other end to the clamping block using support clip (a).
- Extend the helical spring to around three times its original length by moving the clamping block.
- Check to make sure that the oscillation lever is approximately horizontal, and vary the size of the elastic loop as necessary.
- Connect the function generator to the 12 V output of the transformer.
- On the function generator, set the output voltage  $U = 3 V_p$ , frequency range "x 10 Hz" and signal form "~"

**Carrying out the experiment**

*Note: The standing waves generated in this system have an oscillation node at the top coil winding. No node can form at the bottom end of the helical spring because this is where the oscillation is excited.*

**Setup**

Set up the experiment as shown in Fig. 1.

- Connect the stand bases using the short stand rod.
- Mount the long stand rod in the stand base.
- Attach the pointer and clamping block to the stand rod.
- Attach the STE motor (b) in the stand base using the cables and connect it to the function generator (see Fig. 1).
- Measure the length of the unextended helical spring.
- Using a piece of rubber string 15–20 cm long, tie a loop which passes through the eyelet of the oscillation lever and around the short stand rod so that the oscillation lever is approximately horizontal when the helical spring is tensioned.

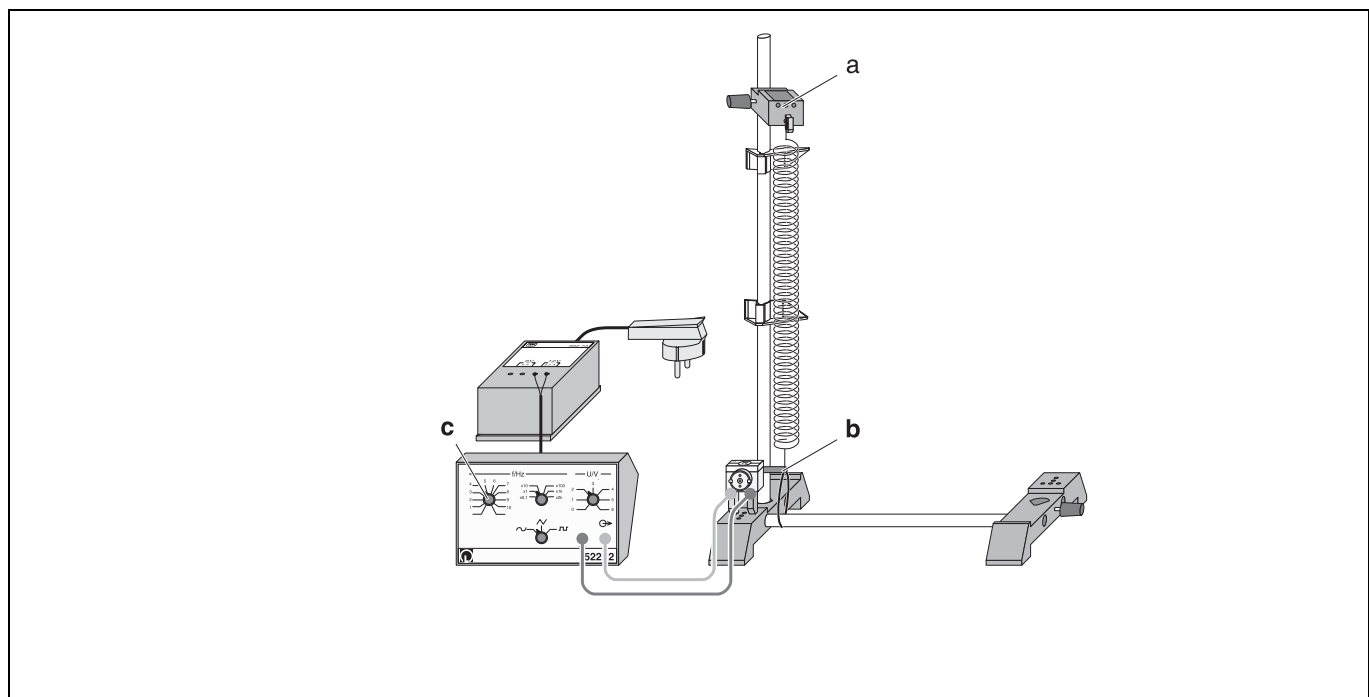
**First experiment with spring 1:**

- Starting from the lowest frequency range, slowly increase the frequency  $f$  and carefully seek those frequencies at which standing waves form; read off the frequencies from the scale of frequency control knob (c).

**Second experiment with spring 1:**

- Once again, generate the standing wave with two oscillation antinodes (three oscillation nodes), mark the positions of the top and bottom oscillation nodes using the pointers and measure the wavelength  $\lambda$  as the distance between these oscillation nodes using the tape measure.

Fig. 1 Experiment setup for exciting standing longitudinal waves in a helical spring



- While maintaining a constant frequency, extend the helical spring further by moving the clamping block and observe how the standing wave persists.
- Once again, measure the distance between the top and bottom oscillation nodes.

#### Experiments with spring 2:

- Place helical spring 2 in the experiment setup, extend it to about twice its original length and repeat the measurements.

### Measuring example and evaluation

#### First experiment with spring 1:

Length unextended: 90 mm

Length extended: 270 mm

Table 1: Frequencies  $f_n$  required for generating standing waves with  $n$  oscillation antinodes.

$n$	$\frac{f}{\text{Hz}}$
1	20
2	28
3	38
4	47
5	57
6	69
7	78
8	90

#### Second experiment with spring 1:

Table 2: Frequency  $f$ , wavelength  $\lambda$  and phase velocity for two oscillation antinodes as a function of the spring length  $s$ .

$\frac{s}{\text{mm}}$	270	300
$\frac{f}{\text{Hz}}$	28	28
$\frac{\lambda}{\text{mm}}$	190	209
$\frac{v}{\text{m} \cdot \text{s}^{-1}}$	5.3	5.9

#### First experiment with spring 2:

Length unextended: 64 mm

Length extended: 128 mm

Table 3: Frequencies  $f_n$  required for generating standing waves with  $n$  oscillation antinodes.

$n$	$\frac{f}{\text{Hz}}$
1	37
2	55
3	73
4	90

#### Second experiment with spring 2:

Table 4: Frequency  $f$ , wavelength  $\lambda$  and phase velocity for two oscillation antinodes as a function of the spring length  $s$ .

$\frac{s}{\text{mm}}$	128	144
$\frac{f}{\text{Hz}}$	55	55
$\frac{\lambda}{\text{mm}}$	93	105
$\frac{v}{\text{m} \cdot \text{s}^{-1}}$	5.1	5.8

### Results

Standing waves can be generated in a helical spring fastened at both ends. The number of oscillation antinodes increases with the frequency.

When the helical spring is extended further, the stationary oscillation state remains the same when the excitation frequency  $f$  is unchanged. The wavelength  $\lambda$  and the phase velocity increase in proportion to the extension.

### Additional information

The excitation frequency set on the frequency generator is affected by feedback from the STE motor. Consequently, a stroboscope (e.g. 451 281) is recommended to precisely determine the oscillation frequency of the string.

