Mechanics

Oscillations

Simple and compound pendulum

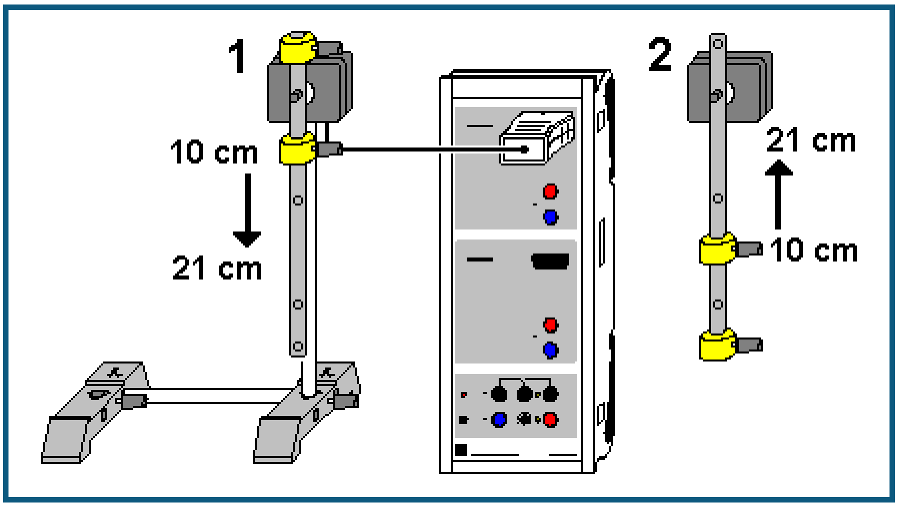
Determination of the acceleration due to gravity on earth by means of a rod pendulum

Description from CASSY Lab 2

For loading examples and settings, please use the CASSY Lab 2 help.

CASSY Lab 2 (2013-06-07)

Determination of the acceleration due to gravity on earth by means of a reversible pendulum



can also be carried out with Pocket-CASSY

Experiment description

For a physical pendulum with small deflections, the oscillation period is given by

$T = 2\pi \cdot \mathrm{sqrt}(l_r/g)$

with the reduced pendulum length $l_r = J/ms$. If the reduced pendulum length l_r and the oscillation period T are known, this can be used for calculating the acceleration due to gravity $g = l_r \cdot 4\pi^2/T^2$.

Often the reduced pendulum length cannot be determined with the desired precision if the precise determination of the moment of inertia or of the center of gravity are difficult. With the reversible pendulum, the mass distribution is modified in such a way that the oscillation period is identical for both rotational axes. From this it can be concluded that the reduced pendulum length l_r corresponds to the distance between the two axes and therefore is known to a high degree of precision.

According to Steiner's theorem, $J = J_S + ms^2$ with J_S being the moment of inertia of the pendulum with respect to the axis through the center of gravity and s being the distance between the center of gravity and rotational axes. The reduced pendulum length is therefore

$l_r = J/ms = J_S/ms + s$.

The second rotational axis is now located on the other side of the center of gravity and, with the same oscillation period and the same reduced pendulum length, is at a distance x from the center of gravity. In this case

$l_r = J_S/mx + x$.

If the equation is rearranged to give the value for x, $x = l_r - s$. The distance between the two rotational axes s+x therefore precisely corresponds to the reduced pendulum length l_r.

Because the oscillation period T can be determined precisely, the reversible pendulum is very suitable for the determination of the value of the earth's acceleration g.

Equipment list

1	Sensor-CASSY	524 010 or 524 013
1	CASSY Lab 2	524 220
1	Rotary motion sensor S	524 082
1	Physical pendulum	346 20
1	Stand rod, 25 cm, d = 10 mm	301 26
2	Stand bases MF	301 21
1	PC with Windows XP/Vista/7/8	

Experiment setup (see drawing)

The pendulum is screwed on the axle of the rotary motion sensor and the mass is attached to the pendulum as shown in (1).

The rod of the pendulum is marked starting from the top in 1 cm steps using a pencil. A range from 10 cm to approximately 21 cm is adequate for this.

Carrying out the experiment

- Load settings
- Attach the variable pendulum mass at the position x = 10 cm and deflect by approximately 10°.
- Once the value displayed for the oscillation period T_{A1} has settled to a constant value and the amplitude α_{A1} has fallen to approximately 5°, record the measuring value by pressing and enter the position in column x (click at the cell in the table with the mouse)
- Push the pendulum mass 1 cm down and measure again. Repeat until x = 21 cm is reached.
- Move the mass back to x = 10 cm and change the suspension point of the pendulum as shown in (2); reverse the pendulum
- Select **Measurement → Append new Measurement Series**.
- Again deflect the pendulum by approximately 10° and wait until the value displayed for the oscillation period T_{A1} has settled to a constant and the amplitude α_{A1} has fallen to approximately 5°, record the measuring value by pressing and enter the position in column x (click on the cell in the table with the mouse)
- Push the pendulum mass 1 cm **upwards** and measure again. Repeat until x = 21 cm is reached.

Evaluation

In the graphic display, two intersections of the oscillation period curves can be seen. In both intersection the period of oscillation and therefore the reduced pendulum length are equal. It corresponds to the displacement of the two axes of rotation, that is l_r = 0.20 m.

By means of a horizontal mark the corresponding period can be determined. In this example it is T = 0.898 s. This results in an earth's acceleration due to gravity of $g = l_r \cdot 4\pi^2/T^2 = 7.896\ \text{m}/T^2 = 9.79\ \text{m/s}^2$.

Alternatively, the earth's acceleration due to gravity can be found with somewhat higher resolution in diagram **g**.

Comments about the measuring error

In addition to the manufacturing error in the rod, which will appear as an error in the reduced pendulum length l_r (approximately Δl_r = ±0.1 mm, that is Δg = ±0.005 m/s^2), an error in the oscillation period T is to be considered. Besides the simple measuring fault (in this case approximately $\Delta T = \pm 0.001 \cdot T$, corresponding to Δg = ±0.02 m/s^2) there is also a systematic fault. As already demonstrated in the experiment Dependency of the period of the oscillation on the amplitude, the oscillation period shows a weak dependency on the amplitude. For the 5° amplitude this systematic error is $\Delta T = +0.0005 \cdot T$, corresponding to Δg = -0.01 m/s^2. With even smaller amplitudes the determination of the oscillation period by means of the rotary motion sensor will become unreliable. For greater amplitudes, this systematic fault will soon exceed the normal measuring fault (for 10° amplitude it is $\Delta T = +0.002 \cdot T$, corresponding to Δg = -0.04 m/s^2)