Determining the gravitational acceleration with a simple pendulum

Objects of the experiment

- Determination of the period of the pendulum as function of wire length.
- Determination of the period of the pendulum as function of the angle of deflection.
- Determination of the gravitational acceleration with a mathematic pendulum.

Introduction

Oscillations (and wave) phenomena are well known due to their presence everywhere in nature and technique. Their investigation is thus both from experimental point of view as from theoretical point of view an important topic which allows not only to demonstrate the fundamental methods and concepts of physics. They introduce also into the mutual interaction between theoretical and experimental physics at an elementary level.

The mathematic pendulum is an outstanding example among various types of mechanical oscillation models which introduces into basic measurement techniques for time and length (angle). For the investigation of other mechanical oscillation models see experiments compound pendulum (i.e. reversible pendulum: P1.5.1.2 to P1.5.1.6), the spring pendulum (P1.5.2.1 or the CASSY experiments with the Laser Motion Sensor S: PCP1.5.1 to PCP1.5.5) and the rotary oscillations (P1.5.3.1 to P1.5.3.4) where all important types of oscillations (free, forced and chaotic oscillations including the influence of friction) can be studied.

Principles

The simple mathematic pendulum is understood to be a point-shaped mass m suspended on a mass-less wire (thread) of length L. Excluding further any frictional forces the motion of the mass can be described by this theoretical approach according to the Newton's principles as follows:

$$J\frac{d^2\varphi}{dt^2} + D\cdot\sin\varphi = 0 \qquad \text{(I)}$$

$J = m\cdot L^2$: moment of inertia related to the point of suspension

$D = m\cdot g\cdot L$ directional moment

g: gravitational acceleration

φ: angle of deflection

m: mass

The solution of equation (I) gives for small deflections ($\sin\varphi \approx \varphi$) that the mass oscillates under the influence of gravity with the period:

$$T = 2\cdot\pi\sqrt{\frac{J}{D}} = 2\cdot\pi\sqrt{\frac{L}{g}} \qquad \text{(II)}$$

Thus a mathematic pendulum can be used to determine the gravitational acceleration g precisely by measuring the oscillation period T and the wire length L of the pendulum.

Oscillations of the pendulum are also a standard example for introducing the energy concept. The energy equation can be obtained by integrating equation (I) in respect to the time t:

$$L^2\left(\frac{d\varphi}{dt}\right)^2 + 2\,g\,L\,(1-\cos\varphi) = E_{kin} + E_{pot} = E_0 = \text{const} \qquad \text{(III)}$$

E_{kin}: kinetic energy

E_{pot}: kinetic energy

E_0: total energy

The angular velocity vanishes at the reversal point for $\varphi = \alpha$ and one obtains for the potential energy at an reversal point:

$$E_0 = 2\,g\,L\,(1-\cos\alpha) \qquad \text{(IV)}$$

Substituting equation (III) into equation (IV) allows to determine the oscillation period for larger deflection angles φ:

$$\frac{T}{4} = \sqrt{\frac{L}{g}}\int_0^\alpha \frac{d\varphi}{\cos\varphi - \cos\alpha}$$

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With k = sinα/2 the oscillation period can be determined as

$$T = 4\sqrt{\frac{L}{g}}\int_0^{\pi/2}\frac{d\varphi}{\sqrt{1-k^2\sin^2\varphi}} = 4\sqrt{\frac{L}{g}}\cdot K(k)$$

Where K(k) is the complete 1st order elliptic integral. The development of the series for K(k) gives for the oscillation period:

$$T = 2\cdot\pi\sqrt{\frac{L}{g}}\left(1+\frac{1}{4}\sin^2\frac{\varphi}{2}+\cdots\right) \qquad \text{(V)}$$

For small values of the deflection angle φ, i.e φ ≤ 7°, the oscillation period is equal to the result of equation (II) − which is derived from equation (I) by regarding all forces acting on the mass m.

In this experiment a steel ball is suspended by a steel wire. (Alternatively, the fishing line (309 48) may be used in place of the steel wire.) The pendulum suspension comprises a screw hook with a turned socket at its end, and a ring fitted with a needle (point facing inwards) and a bore-hole diametrically opposite to it. The needle is inserted into the socket of the screw hook thus giving a suspension with very little friction.

As the mass of the steel ball is much greater than the mass of the steel wire and due to its friction-less suspension the pendulum can be considered to be a close approximation of the theoretical model of a mathematic pendulum.

Multiple oscillations are recorded to improve the measuring accuracy of the time measurements. For the gravitational acceleration the error then depends essentially on the accuracy with which the length of the pendulum is determined.

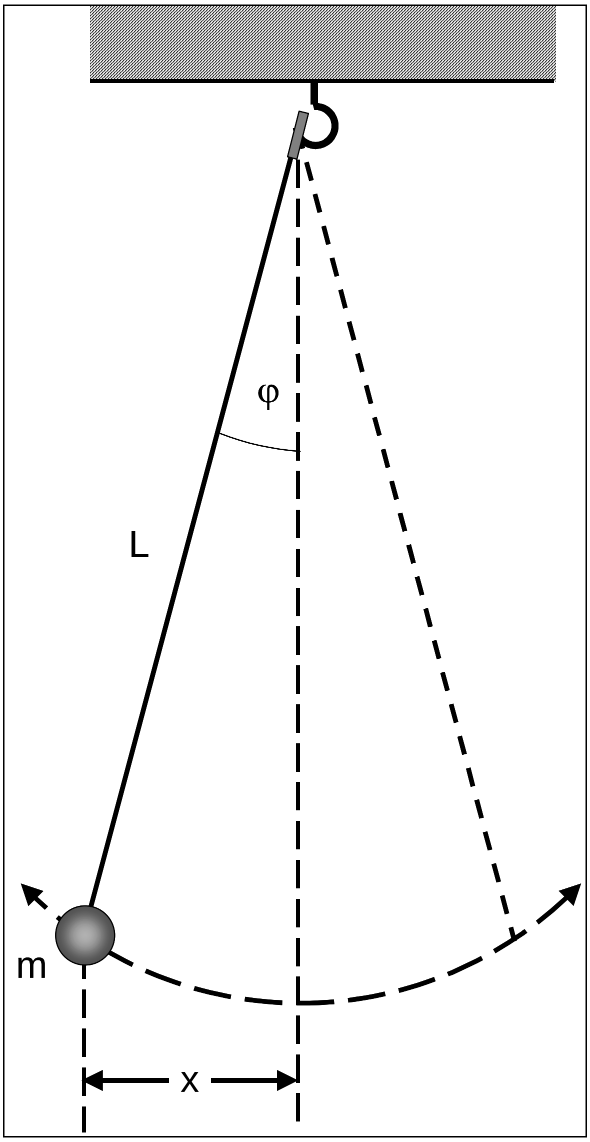

Fig. 1: Experimental setup for measuring the oscillation period T of a mathematic pendulum as function of deflection angle φ and wire length L.

Apparatus

1 Ball with pendulum suspension......................... 346 39
1 Steel tape measure, 2 m................................... 311 77
1 Stop clock 313 07

Setup

Firstly, the hook of the pendulum suspension have to be screwed into a ceiling ledge or by using a dowel set into the ceiling at an appropriate place. It is recommended a place close to a wall to allow an easy determination of the deflection angle.

One end of the steel wire is fitted on the knurled-head screw. After measuring the required length of the pendulum the other end have to be fastened to the ring by threading it several times through the bore hole and twisting it several times by means of a pair of pliers. In place of the steel wire you may use a fishing line (309 48).

After screwing the knurled-head screw into the ball insert the needle in the socket of the hook screwed into the ceiling. The length of the pendulum can be adjusted by either tightening or loosening the screw on the ball.

Safety notes

For large deflection angles trace the area in which the steel ball oscillates to avoid unwanted impacts with operators or observers of the experiment.

Carrying out the experiment

a) Measuring the oscillation period as function of deflection

- Chose the pendulum length greater than 1.0 m.
- Measure the time for at least 10 periods for several angles of deflection.
  To determine the deflection angle φ it is recommended to position the steel tape measure on the floor underneath the pendulum to allow an accurate measurement of the deflection x (see Fig 1.). The angle of deflection is then given by the trigonometric relation:

$$\varphi = \arcsin\frac{x}{L} \qquad \text{(VI)}$$

b) Measuring the oscillation period as function of length

- As a Start chose the length of the wire greater than 1.0 m.
- Measure the time for at least 10 periods to minimize the error of the measurement.
- Measure the length L of the pendulum.
- Repeat the measurement for different lengths of the pendulum.
  It is recommended to vary the wire length between 0.8 m to 2.0 m.

Measuring example

a) Measuring the oscillation period as function of deflection

Table. 1: Oscillation period T (average over 10 oscillations) as function of the deflection x for a wire length of L = 2.05 m. The deflection angle φ is calculated according equation (VI).

$\frac{x}{m}$	$\frac{\varphi}{°\,deg}$	$\frac{T}{s}$
0.10	2,8	2,83
0.20	5,6	2.83
0.30	8,4	2.83
0.40	11,3	2.84
0.50	14,1	2.84
0.60	17,0	2.85
0.80	23,0	2.86
1.00	29,2	2.88
1.20	35,8	2.89
1.35	41,2	2.93
1.50	47,0	2.96
1.70	56,0	3.00

b) Measuring the oscillation period as function of length

Table. 2: Oscillation period T (average over 10 oscillations) as function of the length L (determined for small deflections).

$\frac{L}{m}$	$\frac{T}{s}$
0.80	1.77
1.00	2.10
1.25	2.24
1.50	2.44
1.75	2.66
2.05	2.84

Evaluation and results

Fig 2. summarizes the result of Table 1: The period T is proportional to $\sin^2(\varphi/2)$ in accordance with equation (V).

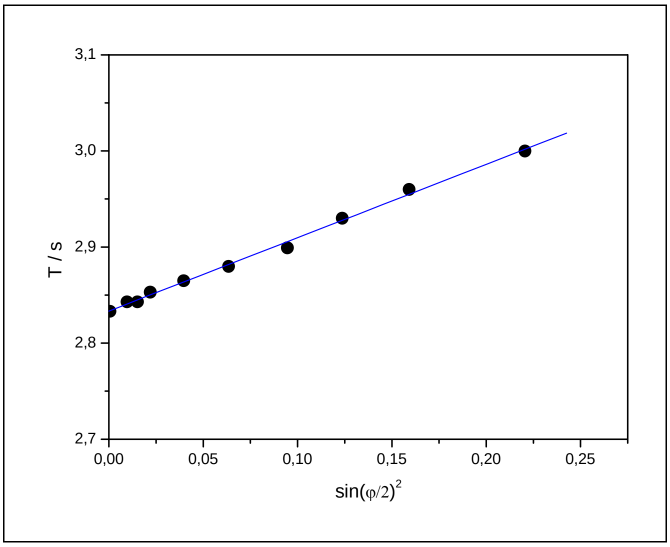

Fig. 2: Period T as function deflection angle for pendulum with length L = 2.05 m. For a clear data evaluation the values of T are plotted versus $\sin^2(\varphi/2)$. The solid line corresponds to a fit according equation (V).

Fig. 3 summarizes the result of Table 2. Between the period T and the length L the following relationship is confirmed:

$$T \sim \sqrt{L}$$

Form the linear regression results follows the gravitational acceleration to (Fig. 3):

$$g = 9.82\,\frac{m}{s^2}$$

$$g = 9.81\,\frac{m}{s^2}$$ (Literature value for Germany)

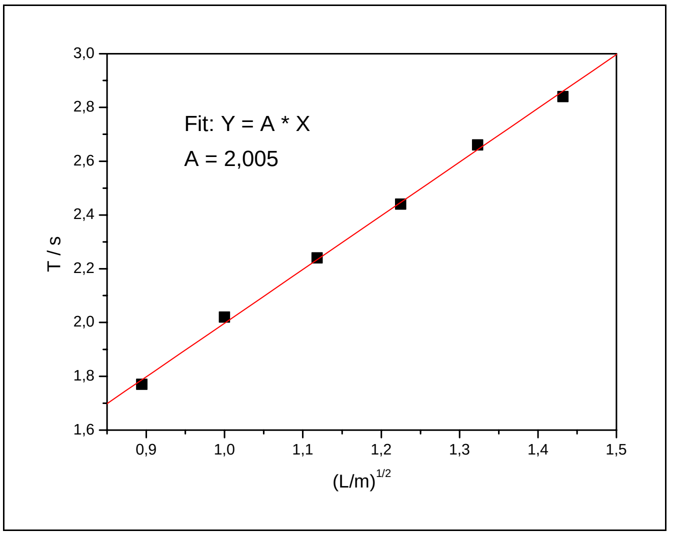

Fig. 3: Period T (determined for small deflection angles φ) as function of the length L of the pendulum. The solid line corresponds to a fit according equation (II).

Supplementary information

The mathematic pendulum was investigated by Galilei to determine the period T. He found that the oscillation period does not depended on the mass of the pendulum. This is in agreement with equations (II) and (V). The pendulum can be used also to demonstrate the independence of the period from its mass.