Maxwell's wheel

Objects of the experiment

- Introducing the concept of conservation of energy
- Measuring the transformation of potential energy into translational and rotational energy
- Determination of moment of inertia of the Maxwell wheel

Principles

Maxwell's wheel is used to demonstrate the conservation of mechanical energy. When the wheel is rotated by hand to the top and released, its potential energy E_{pot} turns into kinetic energy E_{rot} + E_{trans} (rotation and translation) as it falls.

The total energy E of the system is constant:

$$E = E_{pot} + E_{trans} + E_{rot} \qquad (1)$$

or

$$E = m \cdot g \cdot h + \frac{m}{2} v^2 + \frac{I}{2} \omega^2 \qquad (2)$$

where m is the mass and I the inertia of the wheel, h its height position, v its velocity and ω its angular velocity. The acceleration by gravity is denoted as g.

With the estimation, that there is standstill at the beginning (v = 0 and ω = 0) and the motion is in downward (i.e. negative) direction you can rewrite (2) as

$$m \cdot g \cdot h = \frac{m}{2} v^2 + \frac{I}{2} \omega^2 \qquad (3)$$

With the radius of the spindle r you can calculate v according to:

$$v = \omega \cdot r \qquad (4)$$

With (4) in (3) you can determine the inertia of the wheel:

$$I = mr^2 \left(\frac{2gh}{v^2} - 1 \right) \qquad (5)$$

while m = 450 g, r = 3 mm and g = 9.81 m/s^2.

Apparatus

1 Maxwell's wheel	331 22
1 Forked light barrier	337 46
1 Multi-core cable, l = 1.5 m	501 16
1 Counter S	575 471
1 Holding magnet adapter with a release mech.	336 25
1 Scale with Pointers	311 23
1 Saddle base	300 11
1 Support block	301 25
2 Stand base MF	301 21
2 Stand rod, 50 cm	301 27
2 Stand rod, 100 cm	300 44
4 Leybold multiclamp	301 01

Setup

Set up the equipment according to Fig. 1. First assemble the frame with the light barrier using the stand material. Then attach the rod of the Maxwell wheel, in such a way, that the wheel axle is level and the strings are even on both sides. You can reach this best by some up and down movements before starting the experiment.

Attach the key switch using the support block.

Fix the scale to the saddle base. The upper pointer should be adjusted to the axis of the wheel in its highest position. It remains fixed through the complete experiment. The lower pointer is moved according the beam position of the light barrier.

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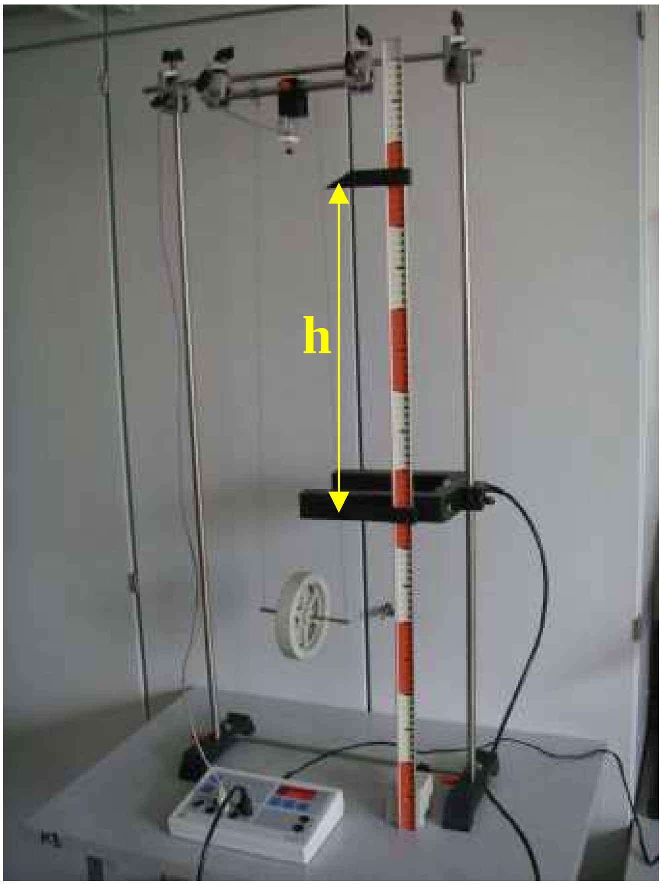

Fig. 1: Experimental setup to examine the conservation of energy using Maxwell's wheel.

Carrying out the experiment

During the experiment one have to measure the time *t* the wheel required between its start position and the position of the light barrier *s* and the velocity of the wheel at this position.

The distance should be varied from 15 cm to 55 cm in steps of 5 cm.

a) Measure the time *t* required for the distance *s* between start to the light barier

- Connect the key switch to port E of the counter. Connect the light barrier to port F.
- Select MODE $t_{E \rightarrow F}$
- Move the wheel to its highest position and let it press the key switch (Fig. 2).
- Press START
- Release the wheel (counter is starting counting)
- After the wheel passed the light barrier the measurement is stopped
- Note down the time *t*

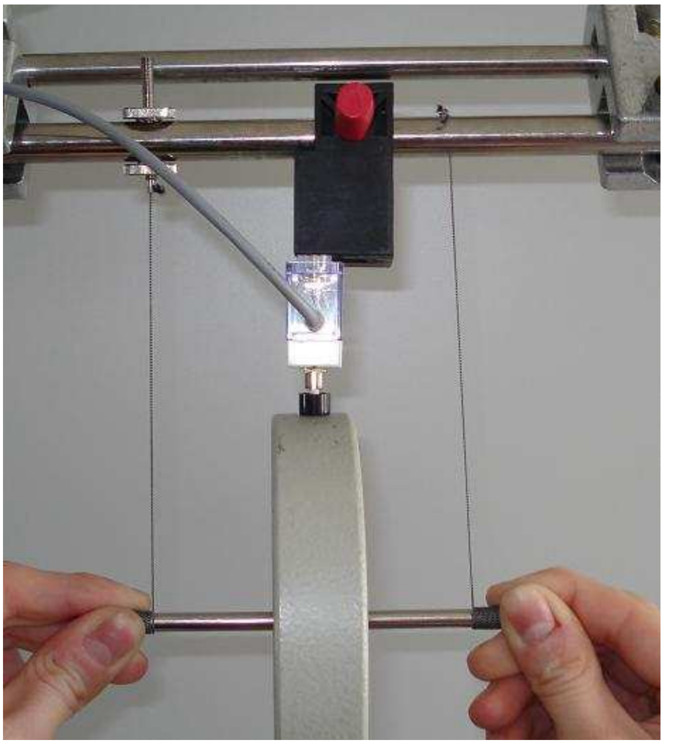

Fig. 2: Wheel in start position

b) Measure the velocity *v* at the light barrier

- Connect the light barrier to port E of the counter
- Select MODE t_E
- Move the wheel to its highest position and let it press the key switch (Fig. 2).
- Press START
- Release the wheel (counter is not starting counting)
- During the wheel passed the light barrier the time Δt is measured
- Note down the time Δt
- Calculate the velocity *v* according to

$$v = \frac{d}{\Delta t} \tag{5}$$

with the diameter of the spindle *d* = 6 mm.

Measuring example

Table 1: Selected distance *h*, measured times *t*, Δt and the calculated velocity *v*

$\frac{h}{\text{cm}}$	$\frac{t}{\text{s}}$	$\frac{\Delta t}{\text{ms}}$	$\frac{v}{\text{m/s}}$
15	2.63	56.50	0.11
20	3.06	47.80	0.13
25	3.47	43.84	0.14
30	3.82	39.35	0.15
35	4.05	36.73	0.16
40	4.45	33.76	0.18
45	4.71	32.13	0.19
50	5.06	30.68	0.20
55	5.18	29.25	0.21

Evaluation and results

a) Examination of dynamics

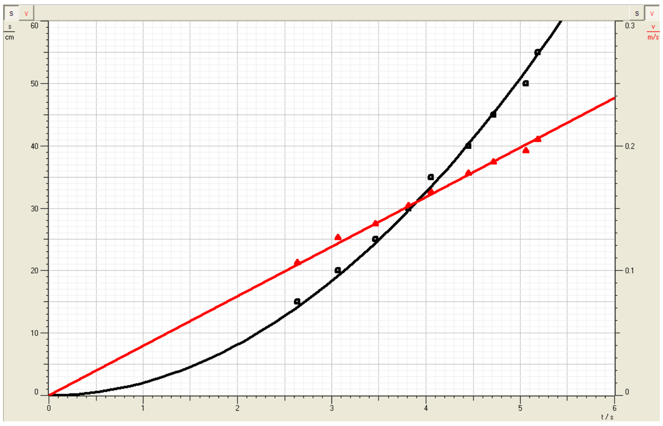

Fig. 3: Distance *s* (black squares) and velocity *v* (red triangles) in dependence of the time *t*

Fig. 3 shows the measured values (Table 1). Note, that the measured times are now used on the x-axis as a scale for the distance and the velocity.

It is obvious, that the values for the distance follows a linear function, while the values for the velocity follows a parabolic function.

So as one can expect $s \sim t^2$ and $v \sim t$ are valid.

b) Determination of the inertia *I*

Inserting the measured values for *h* and *v* in equation (5) we get a value for the inertia *I* for each measurement:

Table 2: Determination of the inertia *I*

$\frac{h}{\text{cm}}$	$\frac{v}{\text{m/s}}$	$\frac{I}{10^{-3}\,\text{kg m}^2}$
15	0.11	1.05
20	0.13	1.00
25	0.14	1.06
30	0.15	1.02
35	0.16	1.04
40	0.18	1.00
45	0.19	1.02
50	0.20	1.03
55	0.21	1.03

From the mean value we can determine

$$I = (1.03 \pm 0.03)\ 10^{-3}\ \text{kg m}^2$$

c) Transformation of energy

Using equation (3) and (4) and the result for the inertia *I* we can calculate the potential energy E_{pot} and the kinetic energy E_{kin} as a sum of the rotational E_{rot} and translational energy E_{trans} (Table 3):

$$E_{kin} = E_{rot} + E_{trans} \qquad (6)$$

Fig. 4 shows the (negative) potential energy and the kinetic energy. They have every time nearly the same value.

Since the values for the translational energy are quite small, we can conclude that most of the potential energy is transformed into rotational energy.

Table 3: Calculation of energies:

$\frac{t}{\text{s}}$	$\frac{E_{pot}}{\text{J}}$	$\frac{E_{kin}}{\text{J}}$	$\frac{E_{rot}}{\text{J}}$	$\frac{E_{trans}}{\text{J}}$
2.63	0,66	0,67	0,65	0,02
3.06	0,88	0,93	0,90	0,03
3.47	1,10	1,10	1,07	0,03
3.82	1,32	1,36	1,33	0,03
4.05	1,55	1,56	1,52	0,04
4.45	1,77	1,85	1,81	0,04
4.71	1,99	2,04	2,00	0,04
5.06	2,21	2,23	2,19	0,04
5.18	2,43	2,45	2,40	0,05

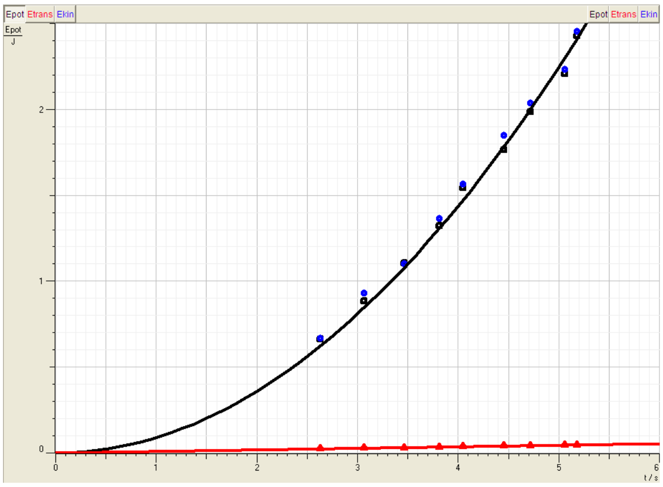

Fig. 4: Potential (black squares), kinetic (blue circles) and translational (red triangles) energies

Supplementary information

The time needed for this experiment needs approx. 20 min for the measurement and 30 min for the evaluation.

As an alternative task one can calculate the gravitational acceleration with a given inertia of $I = 1$ kg m^2.

Also it is possible to measure the drop of height for each oscillation to calculate the loss of energy through friction.

For advanced students it is recommended to discuss the reversion of the direction of motion as a nearly elastic collision.