Point-by-point recording of the projection parabola as a function of the speed and angle of projection

Objects of the experiment

- Determination of the distance as function of inclination angle.
- Determination of the maximum height as function of the inclination angle.

Principles

In the experiment a steel ball of mass m is projected at an angle α to the horizontal with an initial velocity v_0. The motion of the steel ball in the (constant) gravitational field lies in a plane and can be described by the equation (Fig. 1):

$$m\frac{d^2\vec{r}}{dt^2} = m\cdot\begin{pmatrix}0\\-g\end{pmatrix} \qquad \text{(I)}$$

$\vec{r} = \begin{pmatrix}x\\y\end{pmatrix}$: vector of location

m: mass of the steel ball

$\vec{F} = m\cdot\begin{pmatrix}0\\-g\end{pmatrix}$: force acting on the steel ball

Solving equation (I) for the initial conditions

$$\vec{r}(0) = \begin{pmatrix}0\\0\end{pmatrix} \quad \text{and} \quad \vec{v}(0) = \begin{pmatrix}v_0\cdot\cos\alpha\\v_0\cdot\sin\alpha\end{pmatrix}$$

lead to the coordinates of the steel ball as function of time t:

$$x(t) = v_0\cdot\cos\alpha\cdot t$$
$$y(t) = v_0\cdot\sin\alpha\cdot t - \frac{1}{2}g\cdot t^2 \qquad \text{(II)}$$

Form this the range s and the maximum height h is obtained as function of the inclination angle α and the initial velocity v_0:

$$s = \frac{v_0^2}{g}\sin 2\alpha \qquad \text{(III)}$$

$$h = \frac{v_0^2}{2g}\sin^2\alpha \qquad \text{(IV)}$$

In this experiment the range s and the maximum height h as function of the inclination angle α are determined for three different initial velocities v_0.

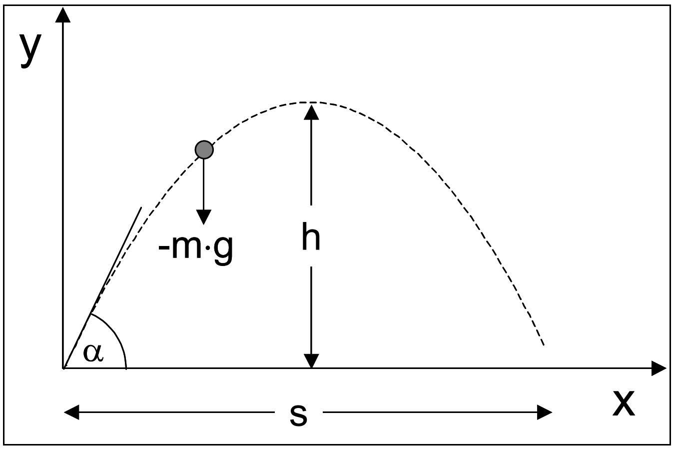

Fig. 1: Movement of a mass in a constant gravitational field. Schematic representation of the chosen coordinate system for the description of the motion with equation (I).

Apparatus

1 Large projection apparatus	336 56
2 Bench clamp	301 06
1 Vertical scale, 1 m	311 22
1 Steel tape measure, 2 m	311 77
1 Saddle base	300 11
1 Laboratory stand II	300 76
1 Tray, 552 x 197 x 48 mm	649 42
1 Bottle of quartz sand, 1 kg	309 00 743

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Safety notes

Please observe the recommendation of the label on the projection apparatus containing the safety notes.

Do not allow a finger to enter the danger region while setting or releasing the projection apparatus.

Be careful not to crush any part of your hand.

Setup

- Mount the projection apparatus as depicted in Fig. 2 on a table.
- Place the tray on the laboratory stand.
- Adjust the height of the surface that either the surface of the sand (method I) or the carbon paper on a white sheet of paper (method II) in the tray is at the same height (10 cm) with the steel ball in the projection apparatus.
- To measure the maximum height h of the trajectory clamp the scale in the saddle base.

Carrying out the experiment

a) Determination of the range as function of inclination angle

- Measure the range s as function of the inclination angle α for a fixed initial velocity v_0.
- Repeat the measurement for the other two possible stages of compression of the projection apparatus, i.e. other two possible initial velocities v_0.

Note: *The points of impact can be recorded either by using sand in the tray (method I) or by using carbon paper on top of a white sheet of paper (method II). For method II it is recommended to secure the white sheet of paper with adhesive tape and number the points of arrival in the sequence of throws (see also instruction sheet 336 56).*

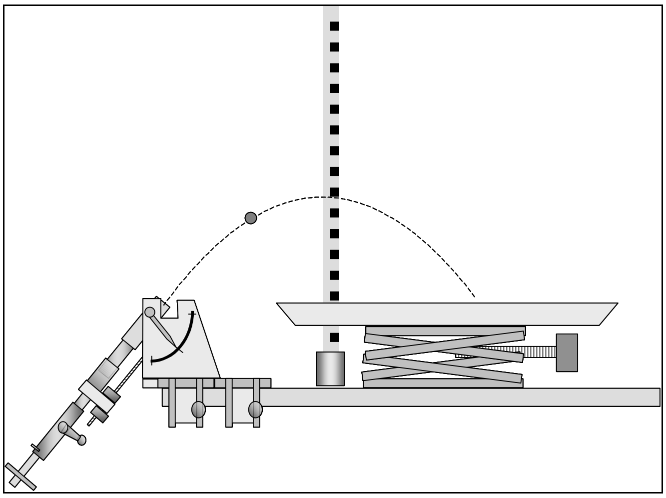

Fig. 2: Schematic diagram of the experimental setup to determine the range and the height as function of the projection angle. Compare instruction sheet 336 56 for further modifications

b) Determination of the height as function of inclination angle

- Measure the maximum height h as function of the inclination angle α for a fixed initial velocity v_0.
- Repeat the measurement for the other two velocities v_0, i.e. the other two positions of the projection apparatus.

Note: *The maximum height h of the of the trajectory can be determined quite well using the movable pointers of the vertical scale. For further information see also instruction sheet 336 56.*

Measuring example

a) Determination of the range as function of inclination angle

Table 1: Range s as function of the inclination angle α for three different initial velocities of the projection apparatus.

$\frac{\alpha}{\text{deg}}$	$\frac{s_1}{\text{m}}$	$\frac{s_2}{\text{m}}$	$\frac{s_3}{\text{m}}$
10	0.130	0.330	0.630
15	0.210	0.430	0.900
20	0.265	0.580	1.180
25	0.320	0.715	1.390
30	0.365	0.825	1.545
35	0.390	0.900	1.670
40	0.410	0.930	1.705
45	0.420	0.940	1.760
50	0.400	0.910	1.710
55	0.375	0.860	1.565
60	0.345	0.800	1.450
65	0.310	0.735	1.320
70	0.245	0.610	1.120
75	0.225	0.470	0.800
80	0.155	0.330	0.540
85	0.085	0.200	0.225

b) Determination of the height as function of inclination angle

Table 2: Maximum height h as function of the inclination angle α for three different initial velocities of the projection apparatus.

$\frac{\alpha}{\text{deg}}$	$\frac{h_1}{\text{m}}$	$\frac{h_2}{\text{m}}$	$\frac{h_3}{\text{m}}$
10	–	0.025	0.035
15	0.0250	0.035	0.075
20	0.030	0.065	0.115
25	0.035	0.105	0.180
30	0.065	0.140	0.235
35	0.080	0.175	0.305
40	0.085	0.213	0.375
45	0.110	0.230	0.460
50	0.130	0.285	0.530
55	0.150	0.320	0.580
60	0.165	0.375	0.640
65	0.185	0.410	0.730
70	0.195	0.422	0.760
75	0.225	0.430	0.825
80	0.235	0.445	0.840
85	0.250	0.485	0.855

Evaluation and results

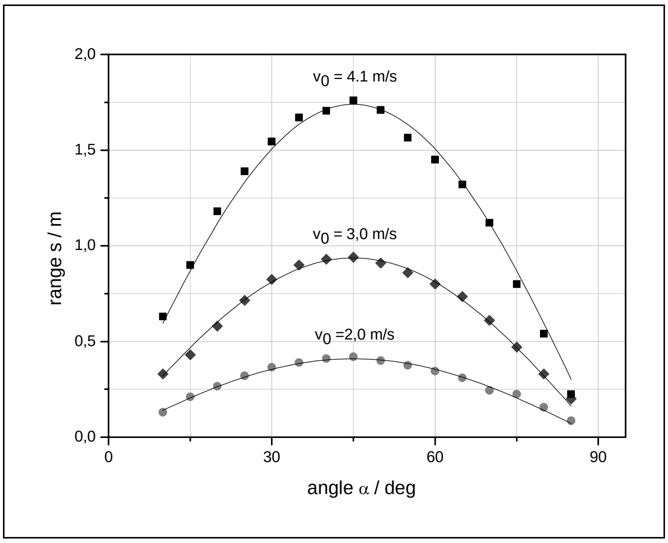

Fig. 3: Range s as function of the inclination angel α for three different initial velocities v_0. Solid lines correspond to a least square fit according equation (III).

Form Fig. 3. the initial velocities v_0 can be determined for α = 45° using equation (III):

$$v_1 = 2.0\frac{\text{m}}{\text{s}}$$

$$v_2 = 3.0\frac{\text{m}}{\text{s}}$$

$$v_3 = 4.1\frac{\text{m}}{\text{s}}$$

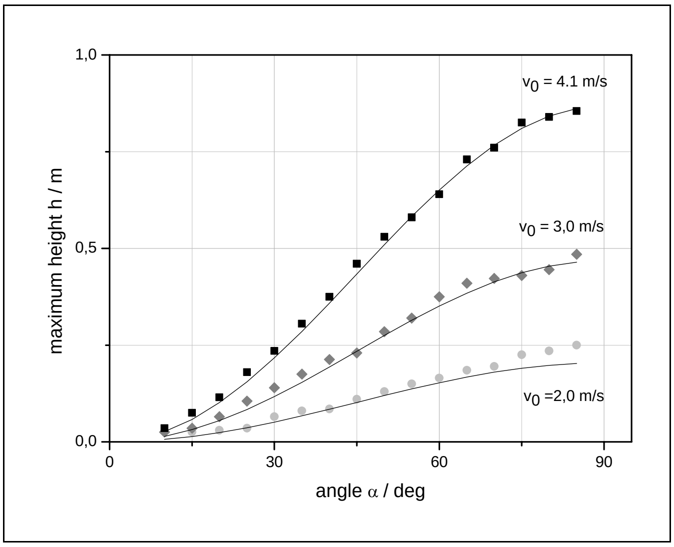

Fig. 4: Maximum height h as function of the inclination angel α for three different initial velocities v_0. Solid lines correspond to a least square fit according equation (IV).

Observable deviations from the parabolic form may be due to friction with the air.

Fig. 3 and Fig. 4 confirm equation (III) and (IV) which have been derived under the assumption of a superposition of a motion with constant velocity in the direction of projection and a vertical falling motion. The trajectory of the steel ball is a parabola whose width and height depend on the inclination angle and the initial velocity.

Supplementary information

The initial velocity v_0 can be measured by using a forked light barrier (337 46). For details of the experimental setup see instruction sheet 336 56. The values measured directly can be compared with the initial velocities found by a least square fit to the experimental data of part a) depicted e.g. in Fig. 3:

Table 3: Comparison of measured initial velocities v_0 with the result of experiment a).

	experiment a)	measured (light barrier)
$\frac{v_1}{m/s}$	2.0	2.1
$\frac{v_2}{m/s}$	3.0	3.1
$\frac{v_3}{m/s}$	4.1	4.0

The measurement of v_0 using the light barrier also allows to show that the initial velocity v_0 is independent of inclination angle α.

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